MODULE - 3

BERNOULLI'S EQUATION

Practical Application Of Bernoulli's Equation.

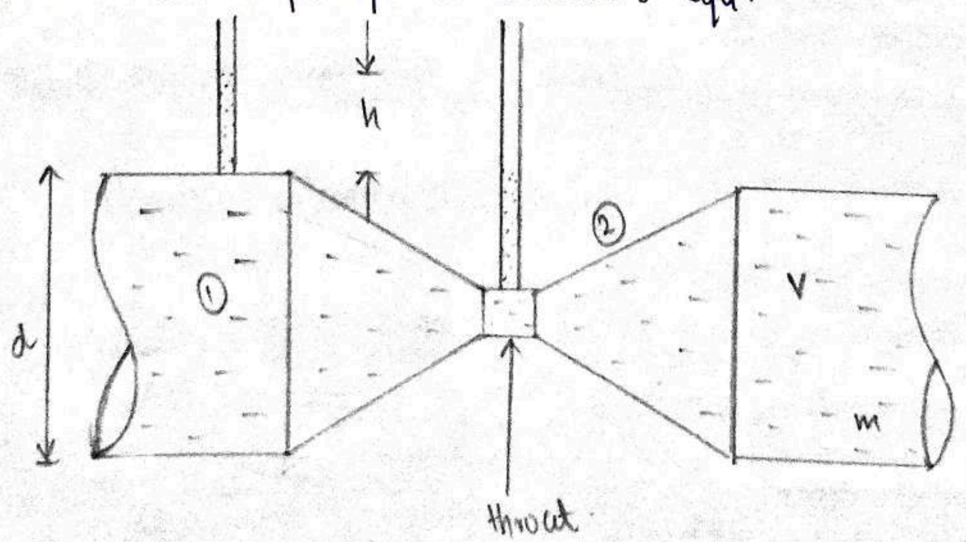
Bernoulli's equ is applied in all problems of incompressible fluid flow where energy consideration are involved. But we shall consider its application to the following measuring devices.

1. Venturimeter
2. Orifice meter
3. Pitot Tube

Venturimeter

It is a device used for measuring the rate of flow of a ~~liquid~~ fluid flowing through a pipe. It consist of 3 parts

(i) Short converging
(ii) throat
(iii) Diverging Part
(iv) It is based on principle of Bernoulli's equ.



$d_1 \longrightarrow$ dia of inlet section ①

$P_1 \longrightarrow$ Pressure at section ①

$V_1 \longrightarrow$ vel of Fluid at section ①

a_1 = area of section ① = $\frac{\pi}{4} d_1^2$

$d_2, P_2, V_2, a_2 \longrightarrow$ At section ②

Applying Bernoulli's equ at ① & ②

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{Z_1} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + \cancel{Z_2}$$

Pipe is Horizontal $Z_1 = Z_2$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

$$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} \quad \text{— ①}$$

$\downarrow$

Diff in Pressure at ① & ② = h

$$h = \frac{P_1 - P_2}{\rho g}$$

Subtituting in ①

$$h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

Applying continuity equ ⓐ at section ① & ②

$$a_1 V_1 = a_2 V_2$$

or

$$V_1 = \frac{a_2 V_2}{a_1}$$

Subitituting in equ

$$h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$h = \frac{V_2^2}{2g} - \frac{\left(\frac{a_2 V_2}{a_1}\right)^2}{2g}$$

$$= \frac{V_2^2}{2g}\left[1 - \frac{a_2^2}{a_1^2}\right]$$

$$= \frac{V_2^2}{2g}\left[\frac{a_1^2 - a_2^2}{a_1^2}\right]$$

$$V_2^2 = 2gh \frac{a_1^2}{a_1^2 - a_2^2}$$

$$V_2 = \sqrt{2gh \frac{a_1^2}{a_1^2 - a_2^2}}$$

$$V_2 = \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$$Q = a_2 V_2$$

$$= a_2 \times \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$$\boxed{Q = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}}$$

$$\boxed{Q_{act} = C_d \times Q}$$

~~Case~~

Value Of ~~Sm~~ 'h' Given By Differential Utube Manometer.

Case 1

Let the differential manometer contains a liquid which is heavier than the liquid flowing through the pipe. Let S_h = specific gravity of heavier liquid.

S_0 = " " of liquid flowing through the pipe.

x = difference of heavier liquid column in utube.

$$h = x\left[\frac{S_h}{S_0} - 1\right]$$

Case 2

If the differential manometer contains a liquid which is lighter than the liquid flowing through pipe, the value of h is given by $h = x\left[1 - \frac{S_L}{S_0}\right]$

S_L - specific gravity of lighter liquid

S_0 - " " of fluid flowing through the pipe

x - difference of lighter liquid columns in utube.

Case 3

Inclined venturimeter with differential utube manometer.

$$h = x\left[\frac{S_h}{S_0} - 1\right]$$

Q) A horizontal venturimeter with inlet and throat diameter 30 and 15 cm resp is used to measure the flow of water the reading of differential manometer connected to the inlet and the throat is 20 cm of mercury. Determine the rate of flow. (Take $C_d = 0.98$)

Ans) $d_1 = 30\,cm$ (Inlet dia)

$d_2 = 15\,cm$ (throat dia)

~~Inlet discharge~~ $a_1 = \frac{\pi}{4} d_1^2 = 706.85\,cm^2$

$a_2 = \frac{\pi}{4} d_2^2 = 176.71\,cm^2$

$C_d = 0.98$

mercury is heavier than water ∴ case I

Reading of differential manometer $= x = 20\,cm$ of mercury.

Difference in pressure $h = x\left[\frac{S_h}{S_o} - 1\right]$

$S_h \rightarrow$ sp gravity of mercury $= 13.6$

$S_o \rightarrow$ " " " water $= 1$

$$h = 20\left[\frac{13.6}{1} - 1\right] = 252\,cm \text{ of water}$$

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \frac{706.85 \times 176.71}{\sqrt{(706.85)^2 - (176.71)^2}} \times \sqrt{2 \times 9.81 \times 252}$$

$$Q = 12576.24\,cm^3/s = \frac{12576.24}{1000} = 12.57624\,kl/s$$

Q) An oil of specific gravity 0.8 is flowing through a venturimeter having inlet dia 20 cm and throat dia 10 cm the oil mercury differential manometer shows a reading of 25 cm. Calculate the discharge of oil through the horizontal venturimeter. Take $C_d = 0.98$.

Ans) Inlet dia = 20 cm = d_1

Throat dia = 10 cm = d_2.

$$a_1 = \frac{\pi}{4} d_1^2 = 314.15\ cm^2$$

$$a_2 = \frac{\pi}{4} d_2^2 = 78.53\ cm^2$$

reading of differential manometer = x = 25 cm

Difference in pressure $h = x\left[\frac{S_h}{S_o} - 1\right]$

$S_h \rightarrow$ sp gravity of mercury = 13.6

$S_o \rightarrow$ sp gravity of oil = 0.8

$$h = 25\left[\frac{13.6}{0.8} - 1\right] = 400\ cm \text{ of oil}$$

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \frac{314.15 \times 78.53}{\sqrt{(314.15)^2 - (78.53)^2}} \times \sqrt{2 \times 9.81 \times 400}$$

$$Q = 7041.29\ cm^3/s = \frac{7041.29}{1000} = 7.04129\ l/s$$

Orifice Meter

- It is a device used to measure the rate of flow of a fluid through a pipe
- It is a cheaper device compared to venturimeter.
- It also works on the same principle of venturimeter.
- It consist of a flat circular ~~weigh~~ plate which has a circular sharp edged

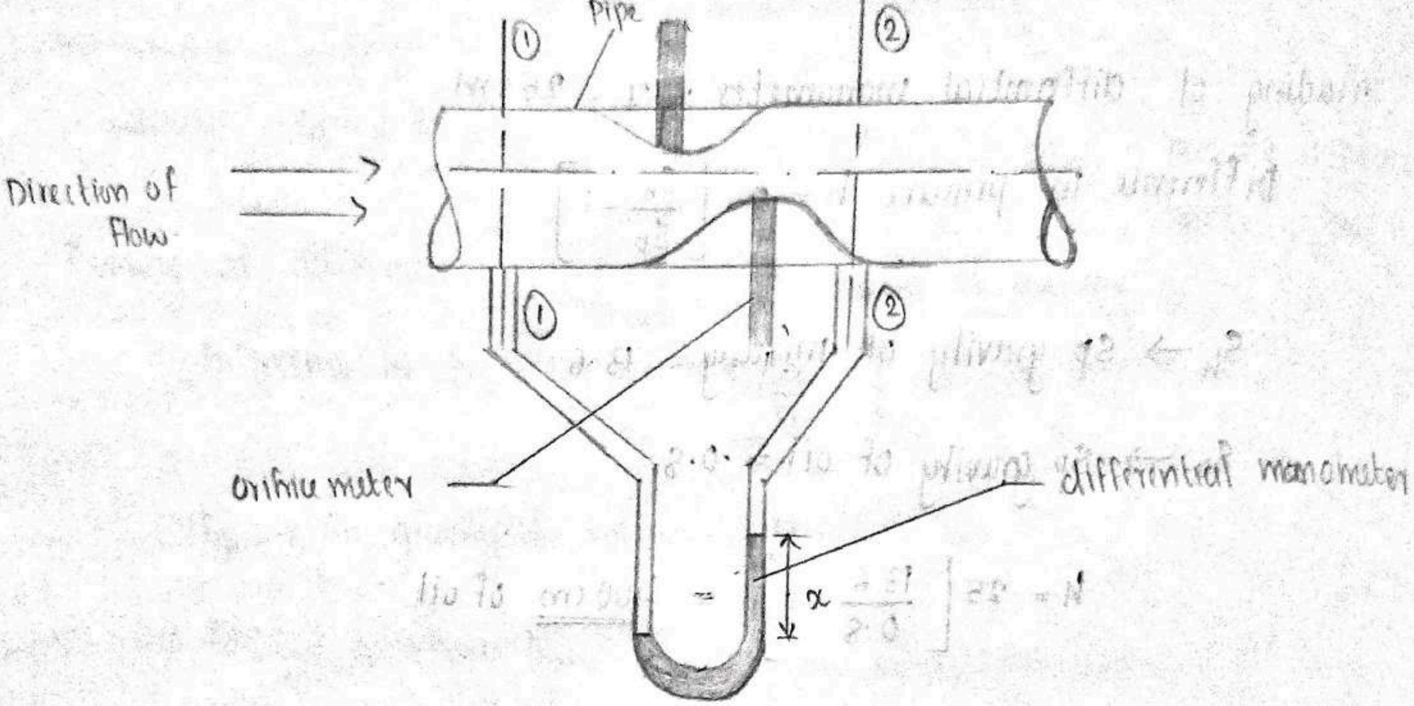
~~hall~~ hole called orifice, which is concentric with pipe.

- The orifice dia generally kept as 0.5 times the dia of pipe.

Let P_1 = pressure at section 1

V_1 = vel at section 1

a_1 = area of pipe at section 1



Q) An orifice meter with an orifice dia 10 cm is inserted in a pipe of 20 cm dia the pressure gauges fitted upstream and downstream of the orifice meter gives readings of 19.62 N/cm² at 9.81 N/cm² resp. Coefficient of discharge for the meter is given as 0.6. Find the discharge of water through the pipe.

Ans)
$$Q = C_d \frac{a_0 a_1}{\sqrt{a_1^2 - a_0^2}} \times \sqrt{2gh}$$

$d_0 = 10\,cm$

$a_0 = \frac{\pi}{4}(d_0)^2 = \underline{78.53\,cm^2} = \underline{0.7853\,m^2}$

$d_1 = 20\,cm$

$a_1 = \frac{\pi}{4}(d_1)^2 = \underline{314.15\,cm^2} = \underline{3.1415\,m^2}$

$$\frac{P_1}{\rho g} = 19.62\ N/cm^2 = 19.62 \times 10^4\ N/m^2$$

$$P_2 = 9.81\ N/cm^2 = 9.81 \times 10^4\ N/m^2.$$

$$\frac{P_1}{\rho g} = \frac{19.62 \times 10^4}{1000 \times 9.81} = \underline{\underline{20\,m \text{ of water}}}$$

$$\frac{P_2}{\rho g} = \frac{9.81 \times 10^4}{1000 \times 9.81} = \underline{\underline{10\,m \text{ of water}}}$$

$$h = \frac{P_1}{\rho g} - \frac{P_2}{\rho g} = 20 - 10 = \underline{\underline{10\,m \text{ of water}}}$$

$$= \underline{\underline{1000\,cm \text{ of water}}}$$

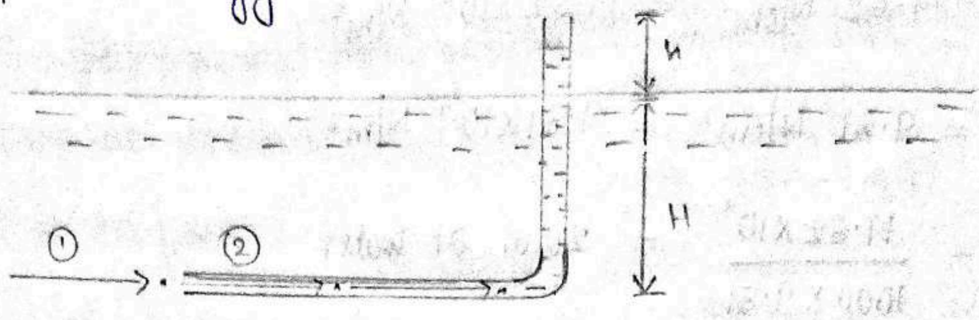
$$Q = 0.6 \frac{78.53 \times 314.15}{\sqrt{78.53^2 - 314.15^2}} \times \sqrt{2 \times 9.81 \times 1000}$$

$$= \underline{\underline{681.62\ cm^3/s}}$$

Pitot Tube

- It is a device used for measuring the vel of flow at any pt in a pipe or a channel.
- It is based on the principle that if the vel of flow at a pt becomes zero. The pressure there is increased due to the conversion of kinetic energy into pressure energy.
- It is a simplest form it consist of a glass tube ~~when~~ went at right angles. The lower end, which is bent 290° is directed in upstream direction as shown in fig. The liquid rises up in the tube due to conversion of kinetic

energy into pressure energy.



Applying Bernoullis equ ① & ②

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{Z_1} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + \cancel{Z_2} \quad \text{—①}$$

$$\left[\therefore Z_1 = Z_2 \ \& \ V_2 = 0\right]$$

$$\frac{P_1}{\rho g} = H.$$

$$\frac{P_2}{\rho g} = H + h$$

Substituting in ①

$$\cancel{H} + \frac{V_1^2}{2g} = h + \cancel{H}$$

$$\frac{V_1^2}{2g} = h$$

$$V_1 = \sqrt{2gh}$$

$$\boxed{V_{act} = C_v \sqrt{2gh}}$$

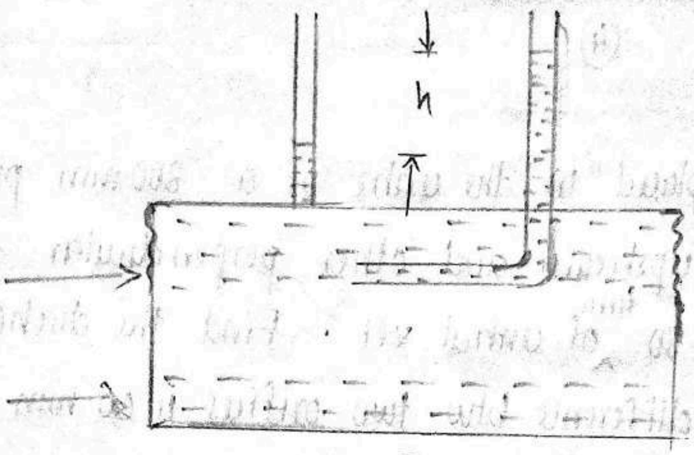
Vel of the flow in a pipe by pitot tube :

~~Vel~~ for finding the vel at any pt in a pipe by pitot tube the following arrangements are adopted.

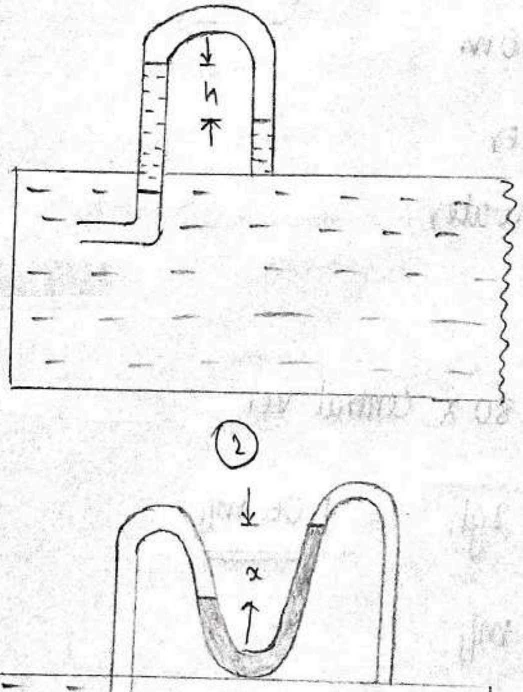
(i) Pitot tube along with vertical ~~piece of a~~ piezometer

(ii) Pitot tube connected with piezometer tube

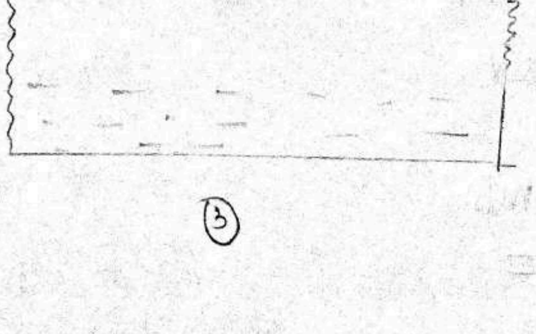
(iii) " " and vertical piezometer ~~tuber~~ connected with a differential u tube manometer.



1



2



3

Pitot tube which consist of 2 circular concentric tube one inside the other with some ~~angul~~ annular space btw them. So the pressure head in the manometer h is given by

$$h = x\left[\frac{S_g}{S_o} - 1\right]$$

(4)

Q) A pitot static tube placed in the centre of a 300 mm pipeline has one orifice, ~~pt in~~ pointing upstream and other perpendicular to it. The mean vel in the pipe is 0.80 times of central vel. Find the discharge through the pipe if the pressure difference btw two orifices is 60 mm of water. Take the coefficient of pitot tube $C_v = 0.98$.

Ans) $d = 300mm = 0.30m$

$h = 60$ mm of water

$= 0.06$ m of water

$C_v = 0.98$

Mean vel $\bar{V} = 0.80 \times$ central vel

central vel $= C_v\sqrt{2gh} = \underline{\underline{1.06 \text{ m/s}}}$

$\bar{V} = 0.85$ m/s.

Q = Area of pipe $\times \bar{V}$

$= \frac{\pi}{4} d^2 \times \bar{V}$

$Q = 0.06 \text{ m}^3/\text{s}$

Q) Find the vel of flow of an oil through a pipe when the differenu of merury in a differential U tube manometer connected to 2 tapping of the pitot tube is 100 mm. Take coefficient of pitot tube 0.98 and sp gravity of 0.8. ~~Find~~

Ans) Given, $x = 100\ mm = 0.1m$ ~~of oil~~

$S_0 = 0.8$

$C_V = 0.98$

$S_g = 13.6$ (mercury sp gravity)

$$h = x\left[\frac{S_g}{S_0} - 1\right] = 1.6\ m \text{ of oil}$$

$$V = C_V\sqrt{2gh} = 5.4\ m/s$$

Flow Through Pipes

- Loss of energy in pipe
- Two types
- Major loss
- Minor loss

Major loss - It can be computed by
- Darcy Weisbach Equation
- Chezys Formula.

Major loss occurs due to friction.

Minor loss - It is due to ① sudden expansion

② Sudden Contraction

③ Pipe fittings

④ Obstruction in pipes

⑤ bend in pipe.

Loss Of Energy Due To Friction

1. ~~Darcey~~ Darcey Weisbach equ

$$h_f = \frac{4FLV^2}{2gD}$$

h_f - loss of h due to friction

F - coefficient of friction

$F - \frac{16}{R_e}$ → reinholt's equ Re - 2000

$$F = \frac{0.079}{Re^{1/4}}$$ Re - 4000 → 10^6

L - length

V - vel of flow

D - dia

2. Chezys Formula

$$V = C\sqrt{mi}$$

C = chezy's constant

$m = \frac{d}{4}$

i = loss of head per unit length of

Q) Find the diameter of pipe of length 2000 m when the rate of flow of water through the pipe is 200 L per sec and to head loss due to friction is 4m. Take the value of C = 50 in Chezy's Formula.

Ans)

$L = 2000\ m$

$Q = 200\ l/s = 0.2\ m^3/s$

$h_f = 4m$

$C = 50$

$D = ?$

$V = \frac{Q}{A} = \frac{0.2}{\frac{\pi}{4}d^2} =$

$V = C\sqrt{mi}$

$\frac{0.2}{\frac{\pi}{4}d^2} = 50\sqrt{\frac{d}{4} \times 2\times10^{-3}}$

$m = \frac{d}{4}$

$d = 553mm$

$i = \frac{h_f}{L}$

$V = 0.2$

$i = 2\times10^{-3}$

$m = \frac{d}{4}$

$i = \frac{h_f}{L} = \frac{4}{2000} = 2\times10^{-3}$

$V = C\sqrt{mi}$

$\frac{V}{C} = \sqrt{mi} \qquad mi = \frac{V^2}{C^2}$

$$\frac{(0.25/d^2)^2}{50^2} = \frac{d}{4} \times 2\times10^{-3}$$

$$0.25^2/d^4 = \frac{d}{4} \times 2\times10^{-3} \times 50^2$$

$$\frac{1}{d^5} = \frac{2\times10^{-3}\times50^2}{0.25^2\times4} = 20$$

$$d^5 = \frac{1}{20} = 0.05$$

Q) Find the head loss due to friction in a pipe of dia 300 mm length 50 m, through which water is flowing of at a vel of 3 m/s using darcy & chezy's formula where $C = 60$. Take ν of water $= 0.01$ stroke

Ans) dia of pipe $= 300\,mm = 0.3\,m$

$l = 50\,m$

$V = 3\,m/s$

$C = 60$

$\nu = 0.01$ stoke $\rightarrow 0.01 \times 10^{-4}$ cm^2/s

darcy's formula

$$h_f = \frac{4fLV^2}{2gD}$$

$$R_e = \frac{V \times d}{\nu} = \frac{3 \times 0.3}{0.01 \times 10^{-4}} = 900000$$

$$f = \frac{0.079}{900000^{1/4}} = 2.564 \times 10^{-3}$$

$$h_f = 1 \times \frac{2.56 \times 50 \times 3^2}{2 \times 9.81 \times 0.3} = \cancel{0.78} \; \underline{0.782\,m}$$

Chezy's formula

$$V = C\sqrt{mi}$$

$$m = \frac{d}{4} = \frac{0.3}{4} = 0.075$$

$V = 3\,m/s$ $\quad i = \frac{h_f}{L}$

$$3 = 60\sqrt{0.075 \times i}$$

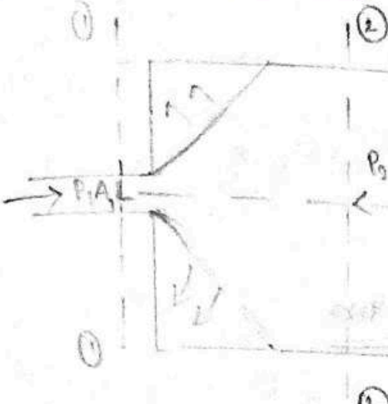
$$\frac{3}{60\sqrt{0.075}} = i$$

$$0.033 = i$$

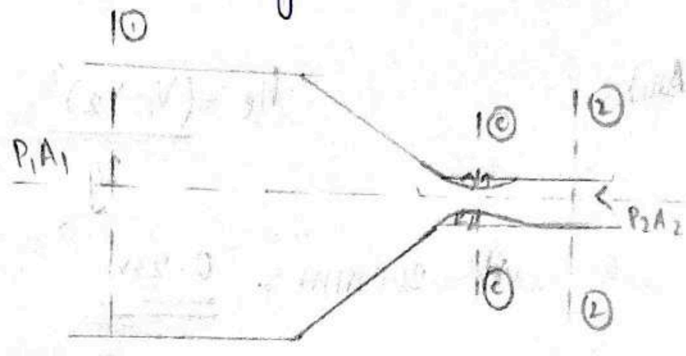
$\cancel{i} \; 0.033 = \frac{h_f}{30}$ $\quad$ $\underline{h_f = 1.65}$

Minor loss

Loss of energy due to change of velocity of following fluid in magnitude or direction is called minor loss of fluid due to sudden enlargment.



$$h_e = \frac{(V_1 - V_2)^2}{2g}$$



$$h_c = \frac{kV_2^2}{2g} \qquad h_c = \frac{0.375 V_2^2}{2g}$$

$$k = \left[\frac{1}{C_c} - 1\right]^2$$

$$h_c = \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2$$

due to contraction

Loss of Head At ~~extreme~~ entrance Of Pipe

$$h_i = 0.5\frac{V^2}{2g}.$$

Loss of head at exit of pipe

$$h_o = \frac{V^2}{2g}$$

Loss of head due to obstruction on pipe.

$$\frac{V^2}{2g}\left(\frac{A}{C_c(A-a)} - 1\right)^2$$

A – Area of pipe

a – Area of obstruction.

Loss of head due to bend in pipe

$$h_b = \frac{kV^2}{2g}$$

Loss of head in Various pipe fittings.

$$\frac{kV^2}{2g}$$

$k \rightarrow$ coefficient of pipe fitting.

Q) Find the loss of head when a pipe of diameter 200mm is suddenly enlarged to a dia of 400mm. The rate of flow of water through the pipe is 250 l/s.

Ans)

$$h_e = \frac{(V_1-V_2)^2}{2g}$$

$d_1 = 200mm = \underline{0.2m}$ $\qquad d_2 = 400mm = \underline{0.4m}$

$$Area_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} \times (0.2)^2 = \cancel{1.816}\ \underline{0.03141\ m^2}$$

$$A_2 = \frac{\pi}{4} d_2^2 = \underline{0.12564\ m^2}$$

$$Q = 250\ l/s = \underline{0.25\ m^3/s}$$

$$V_1 = \frac{Q}{A_1} = \frac{0.25}{0.03141} = \underline{7.96\ m/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.25}{0.12564} = \underline{1.99\ m/s}$$

$$h_e = \frac{(V_1-V_2)^2}{2g} = \frac{(7.96-1.99)^2}{2 \times 9.81} = \underline{1.816\text{m of water}}$$

Q) A horizontal pipe of diameter 500mm is suddenly contracted to a diameter 250mm. The pressure intensities in the large and smaller pipe is given as 13.734 N/cm² and 11.772 N/cm² resp. Find the loss of head due to contraction if $C_c = 0.62$ also determine the rate of flow of water.

Ans) $d_1 = 500\,mm = \underline{\underline{0.5\,m}}$

$d_2 = \cancel{0.25}\ 250\,mm = \underline{\underline{0.25\,m}}$

$A_1 = \frac{\pi}{4} d_1^2 = \underline{\underline{0.19^{63}\,m^2}}$

$A_2 = \frac{\pi}{4} d_2^2 = \underline{\underline{0.04\ m^2}}$

$P_1 = 13.734\ N/cm^2 = 13.734 \times 10^4\ N/m^2$ $\qquad$ $N/cm^2 \to N/m^2$ $\times 10^4$.

$P_2 = 11.772\ N/cm^2 = 11.772 \times 10^4\ N/m^2$

$C_c = 0.62$

$$* \; h_c = \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2$$ equ due to contraction.

continuity equ, we have

$$A_1V_1 = A_2V_2$$

$$h_c = 0.375\,\frac{V_2^2}{2g}$$ $\Rightarrow$ equ when value of C_c is given

$$A_1V_1 = A_2V_2.$$

$$V_1 = \frac{A_2V_2}{A_1} = \cancel{\frac{0.19\,V_2}{0.04}} = \frac{\frac{\pi}{4}d_2^2 V_2}{\frac{\pi}{4}d_1^2} = \frac{0.25^2 V_2^2}{0.5^2}$$

$$V_1 = \frac{V_2}{4}$$

Applying Bernoullis equ

Before and after contraction $\quad \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_c.$

$$Z_1 = Z_2$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_c.$$

$$h_c = \frac{0.375\ V_2^2}{2g}$$

$$V_1 = \frac{V_2}{4}$$

$$\frac{13.734 \times 10^4}{1000 \times 9.81} + \frac{\left(\frac{V_2}{4}\right)^2}{2 \times 9.81} = \frac{11.772 \times 10^4}{1000 \times 9.81} + \frac{V_2^2}{2 \times 9.81} + \frac{0.375 V_2^2}{2 \times 9.81}$$

$$14 + \frac{V_2^2}{16 \times 2 \times 9.81} = 12 + \frac{V_2^2}{19.62} + \cancel{0.0149 V_2^2}\ \frac{0.375 V_2^2}{19.62}$$

$$14 + \frac{V_2^2}{313.92} = 12 + \frac{V_2^2}{19.62} + \frac{0.375 V_2^2}{19.62}.$$

$$\cancel{\frac{4394.88 + V_2^2}{313.92}} = \cancel{\frac{235.44 + V_2^2 + 0.375 V_2^2}{19.62}}.$$

$$14 + \frac{V_2^2}{313.92} = 12 + \frac{1.375\ V_2^2}{19.62}$$

$$14 - 12 = \frac{1.375\ V_2^2}{19.62} - \frac{V_2^2}{313.92}.$$

$$2 = \frac{431.64 V_2^2 - 19.62 V_2^2}{6159.1104}$$

$$412.02\, V_2^2 = 12318.2208$$

$$V_2^2 = 30.042$$

$$V_2 = 5.481\ m/s$$

$$h_c = \frac{0.375 \times 5.481^2}{2 \times 9.81} = 0.5741$$

Q) Water is Flowing through a horizontal pipe of dia 200mm at a vel of 3m/s a circular solid plate of dia 150mm is placed in the pipe to obstruct the flow. Find the loss of head due to obstruction in pipe If $C_c = 0.62$.

Ans) Given

$D = 200mm = 0.2m$

$V = 3m/s$

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (0.2)^2 = 0.031\ m^2$

dia of obstruction, $d = 150mm = 0.15m$

$a = \frac{\pi}{4} \times 0.15^2 = 0.017\ m^2$

$C_c = 0.62$

Loss of head due to obstruction $= \frac{V^2}{2g} \left[\frac{A}{C_c(A-a)} - 1 \right]$

$$= \frac{3^2}{2 \times 9.81} \left[\frac{0.031}{0.62(0.031 - 0.017)} - 1 \right]^2$$

$$= 3.3m$$

<u>Hydraulic Gradient Line And Total Energy Line</u>

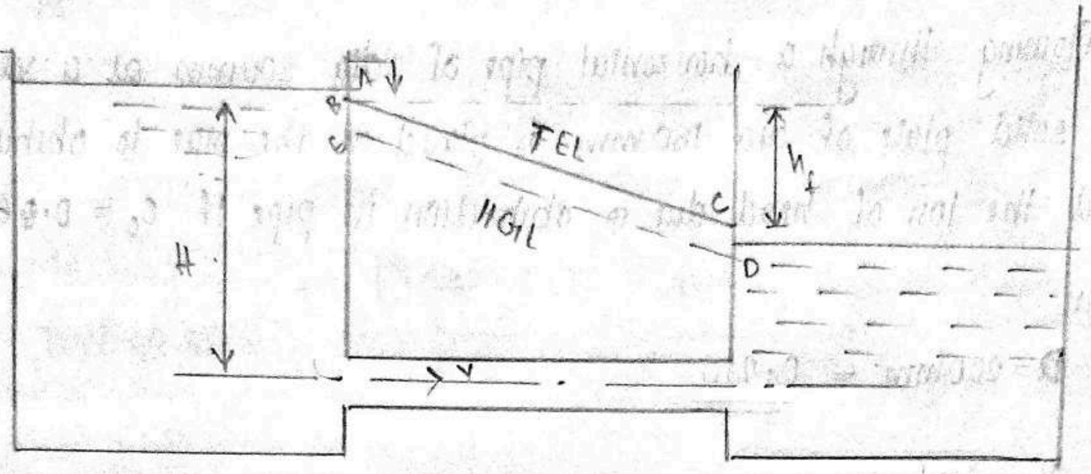
<u>Hydraulic Gradient</u>

It is defined as the line which gives sum of pressure head $\left(\frac{P}{w}\right)$ and datum head

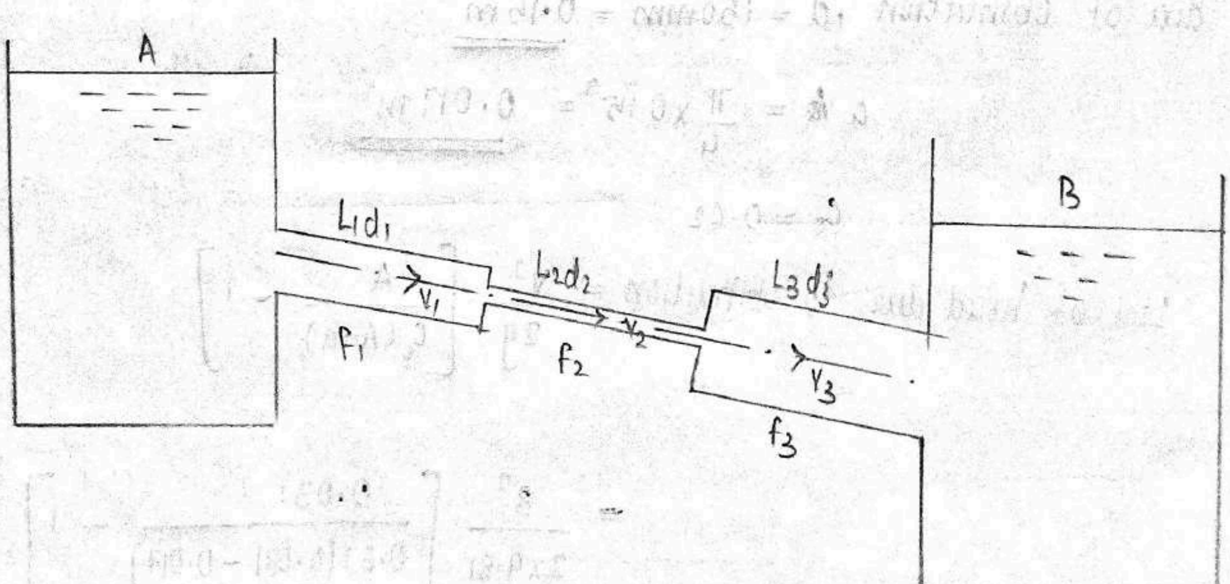
(z) of a flowing fluid in a pipe w.r.t ~~some~~ some reference line

Total Energy Line

It is defined as the line which gives sum of pressure head, datum head, and kinetic head of a flowing fluid in a pipe w.r.t same reference line.



Flow Through Pipe In Series OR Flow Through Compound Pipes



$$Q = A_1V_1 = A_2V_2 = A_3V_3$$

$$H = \frac{4f_1L_1V_1^2}{2gd_1} + \frac{4f_2L_2V_2^2}{2gd_2} + \frac{4f_3L_3V_3^2}{2gd_3}$$

If $f_1 = f_2 = f_3$

$$H = \frac{4f}{2g}\left[\frac{L_1V_1^2}{d_1} + \frac{L_2V_2^2}{d_2} + \frac{L_3V_3^2}{d_3}\right]$$

Total head loss, (i- minor loss) $1=H$

$$H = \left[\underbrace{\frac{0.5V_1^2}{2g}}_{\text{Loss of head entrance}} + \frac{4f_1L_1V_1^2}{2gd_1}\right] + \left[\frac{0.5V_2^2}{2g} + \frac{4f_2L_2V_2^2}{2gd_2}\right] + \left[\overbrace{\frac{(V_2-V_3)^2}{2g}}^{\text{expansion}} + \frac{4f_3L_3V_3^2}{2gd_3} + \overbrace{\frac{V_3^2}{2g}}^{\text{exit}}\right]$$

Difference in water surface level in two tanks which are connected by 3 pipes in series of length 300m, 170m & 210m and of diameters 300mm, 200mm and 400mm resp. is 12m. Determine the rate of flow of water if coefficient of friction are 0.005, 0.0052 and 0.0048 resp. considering

(i) minor losses

(ii) neglecting minor losses.

Ans) $H = 12m$

Pipe 1

$l_1 = 300m$

$d_1 = 300\ mm = \underline{\underline{0.3m}}$

$f_1 = 0.005$

Pipe 2

$l_2 = 170m$

$d_2 = 200mm = \underline{\underline{0.2m}}$

$f_2 = 0.0052$

Pipe 3

$l_3 = \cancel{400m}\ 210m$

$d_3 = 400mm = \underline{\underline{0.4m}}$

$f_3 = 0.0048$

$$Q = A_1V_1 = A_2V_2 = A_3V_3$$

$$V_2 = \frac{A_1V_1}{A_2} = \frac{\frac{\pi}{4}d_1^2V_1}{\frac{\pi}{4}d_2^2} = \frac{0.3^2\ V_1}{0.2^2} = \underline{\underline{2.25V_1}}$$

$$V_3 = \frac{A_1 V_1}{A_3} = \frac{\frac{\pi}{4} d_1^2 V_1}{\frac{\pi}{4} d_3^2} = \frac{0.3^2 V_1}{0.4^2} = \underline{\underline{0.5625\, V_1}}$$

$$\cancel{H = \frac{4 f_1 L_1 V_1^2}{2 g d_1} + \frac{4 f_2 L_2 V_2^2}{2 g d_2} + \frac{4 f_3 L_3 V_3^2}{2 g d_3}}$$

$\cancel{H} \cancel{=}$

$$H = \left[\frac{0.5 V_1^2}{2g} + \frac{4 f_1 L_1 V_1^2}{2 g d_1}\right] + \left[\frac{0.5 V_2^2}{2g} + \frac{4 f_2 L_2 V_2^2}{2 g d_2}\right] + \left[\frac{(V_2 - V_3)^2}{2g} + \frac{4 f_3 L_3 V_3^2}{2 g d_3} + \frac{V_3^2}{2g}\right]$$

$$12 = \left[\frac{0.5 V_1^2}{2 \times 9.81} + \frac{4 \times 0.005 \times 300 \times V_1^2}{2 \times 9.81 \times 0.3}\right] + \left[\frac{0.5 \times (2.25 V_1)^2}{2 \times 9.81} + \frac{4 \times 0.0052 \times 170 \times (2.25 V_1)^2}{2 \times 9.81 \times 0.2}\right]$$

$$+ \left[\frac{(2.25 V_1 - 0.5625 V_1)^2}{2 \times 9.81} + \frac{4 \times 0.0048 \times 210 \times (0.5625 V_1)^2}{2 \times 9.81 \times 0.4} + \frac{(0.5625 V_1)^2}{2 \times 9.81}\right]$$

$\cancel{12} = \cancel{[2 V_1^2 \times 1.044]} +$

$$12 = \frac{V_1^2}{2g}\left[0.5 + 20 + 2.53 + 89.505 + 2.847 + 3.189 + 0.316\right]$$

$$12 = \frac{V_1^2}{2g} \times 118.887$$

$$V_1 = \sqrt{\frac{12 \times 2 \times 9.81}{118.887}} = 1.407 \text{ m/s}$$

$$Q = A_1 V_1$$

$$= \frac{\pi}{4} d_1^2 \times V_1 = \frac{\pi}{4} \times 0.3^2 \times 1.407 = 0.09945 \text{ m}^3/\text{s}$$

(ii) Neglecting Minor loss

$$H = \frac{4 f_1 L_1 V_1^2}{2 g d_1} + \frac{4 f_2 L_2 V_2^2}{2 g d_2} + \frac{4 f_3 L_3 V_3^2}{2 g d_3}$$

$$12 = \frac{V_1^2}{2g}\left[\frac{4 \times 0.005 \times 300}{0.3} + \frac{4 \times 0.0052 \times 170 \times (2.52)^2}{0.2} + \frac{4 \times 0.0048 \times 210 \times (0.5625)^2}{0.4}\right]$$

$$12 = \frac{V_1^2}{2g}\left[20 + 89.505 + 3.189\right]$$

$$12 = \frac{V_1^2}{2g} \times 112.694$$

$$V_1 = \sqrt{\frac{2 \times 9.81 \times 12}{112}}$$

$$Q = A_1 V_1 = 1.445 \times \frac{\pi}{4} (0.3)^2 = 0.1021 \text{ m}^3/\text{s} = 102.1 \text{ l/s}$$

(iii) Flow Through Equivalent Pipe

This is defined as the pipe of uniform diameter having loss of head and discharge equal to the loss of head and discharge of ~~tort~~ compound pipe consisting of several pipes of different length and diameter.

l_1 = length of pipe 1

l_2 = length of pipe 2

l_3 = length of pipe 3

$$\frac{L}{d^5} = \frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}$$

Q) 3 pipes of length 800m, 500m, & 400m and of diameters 500mm, 400mm & 300mm resp. are connected in series. These pipes are replaced by a single pipe of length 1700m. Find diameter of single pipe.

Ans) $l_1 = 800\,m$

$l_2 = 500m$

$l_3 = 400m$

$d_1 = 500mm = 0.5m$

$d_2 = 400mm = 0.4m$

$d_3 = 300mm = 0.3m$

$L = 1700m$

$$\frac{L}{d^5} = \frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}$$

$$\frac{1700}{d^5} = \frac{800}{0.5^5} + \frac{500}{0.4^5} + \frac{400}{0.3^5}$$

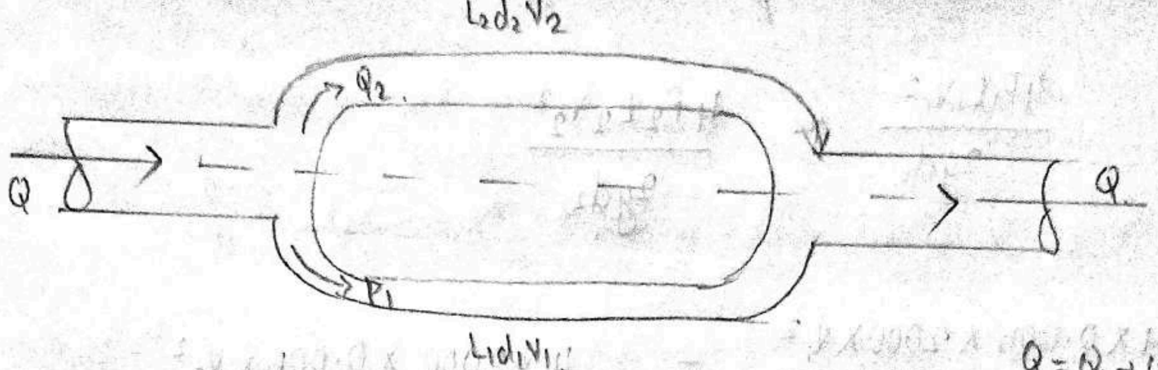
$$d^5 = 7.11 \times 10^{-3}$$

$\sqrt[5]{\ }$ calculator.

$$d = (7.11 \times 10^{-3})^{0.2} = 0.3718 = 371.8\ mm$$

Flow Through Parallel Pipe

consider a main pipe which divides into two or more branches as shown in fig. and again joined together downstream to form a single pipe. Then the branch pipe are said to be connected in parallel. ~~the~~ discharge through the main is increased by connecting pipes in parallel. ~~The disch~~



$Q = Q_1 + Q_2$

$$\frac{4 f_1 L_1 V_1^2}{d_1 \times 2g} = \frac{4 f_2 L_2 V_2^2}{2g d_2}$$

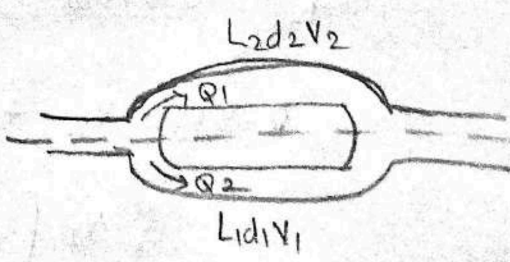
$$f_1 = f_2$$

$$\frac{L_1 V_1^2}{2g d_1} = \frac{L_2 V_2^2}{2g d_2}$$

Q) A main pipe divides into two parallel pipes which again forms one pipe as shown in fig. The length and dia for the 1st parallel pipe are 200$\hat{0}$m and 1m resp. while the length and dia of second parallel pipe are 2000m and ·8m resp. Find the rate of flow in each parallel pipe if total flow in the main pipe is 3 m³/s. The coefficient of friction for each parallel pipe is same and equal to 0·005.

$L_1 = 2000\,m$

$d_1 = 1m$



$L_2 = 2000\,m$

$d_2 = 0.8\,m$

$Q = 3\,m^3/s$

$f_1 = f_2 = f = 0.005$

$Q = Q_1 + Q_2 = 3$ —— (1)

$$\frac{4f_1L_1V_1^2}{2gd_1} = \frac{4f_2L_2V_2^2}{2gd_2}$$

$$\frac{4 \times 0.005 \times 2000 \times V_1^2}{2 \times 9.81 \times 1} = \frac{4 \times 2000 \times 0.005 \times V_2^2}{2 \times 9.81 \times 0.8}$$

$$\frac{V_1^2}{1} = \frac{V_2^2}{0.8}$$

$$V_1 = \frac{V_2}{\sqrt{0.8}} = \frac{V_2}{0.894} \quad \text{—— (2)}$$

$$Q_1 = \frac{\pi}{4} d_1^2 \times V_1 \Rightarrow \frac{\pi}{4} \times (1)^2 \times \frac{V_2}{0.894}$$

$$Q_2 = \frac{\pi}{4} d_2^2 \times V_2 = \frac{\pi}{4} \times (0.8)^2 \times V_2$$

Sub Q_1 & Q_2 in equ (1)

$$Q = \left[\frac{\pi}{4} \times \frac{V_2}{0.894}\right] + \left[\frac{\pi}{4} \times 0.8^2 \times V_2\right] = 3$$

$$0.8785\, V_2 + 0.5026\, V_2 = 3$$

$$V_2\left[0.8785 + 0.5026\right] = 3$$

$V_2 = 2.17$ m/s

Sub the value in equ (2)

$$V_1 = \frac{2.17}{0.894} = 2.47 \text{ m/s}$$

$$Q_1 = \frac{\pi}{4} d_1^2 V_1 \Rightarrow \frac{\pi}{4} \times 1^2 \times 2.427 = 1.906 \text{ m}^3/\text{s}$$

$$Q_2 = \frac{\pi}{4} d_2^2 V_2 \Rightarrow \frac{\pi}{4} \times 0.8^2 \times 2.13 = 1.094 \text{ m}^3/\text{s}.$$

Momentum Equation

It is based on law of conservation of momentum which states that the net force acting on a fluid mass is equal to the change in momentum of flow per unit time with that direction. The force acting on a fluid mass 'm' is given by newton second law of motion.

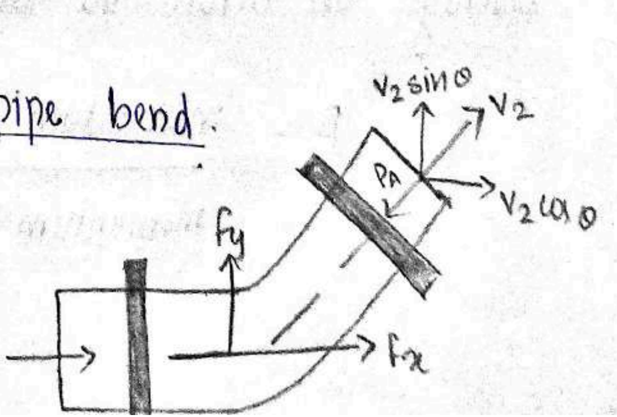
$$F = m \times a$$

$$F = m\frac{dv}{dt}$$

$$F = \frac{d}{dt}(mv)$$

$$F dt = d(mv)$$

Force Exerted by a Flowing fluid on a pipe bend.



V_1 = vel at section (1)

P_1 = pressure intensity at section

A_1 = area of cross section of pipe section at ①

$V_2 P_2 A_2$ → corresponding vel, pressure, area of cross section ②

Net force acting on fluid in 'x' direction of x = rate of change of momentum in 'x' direction.

$$P_1A_1 - P_2A_2\cos\theta - F_x = \text{(Mass per sec) (change of vel)}$$

$$= \rho Q \text{ (Final vel in the x direction)}$$

$$P_1A_1 - P_2A_2\cos\theta - F_x = \rho Q\,(V_2\cos\theta - V_1)$$

$$F_x = \rho Q[V_2\cos\theta - V_1] + P_1A_1 - P_2A_2\cos\theta$$

Similarlly the momentum equ in y direction gives

$$0 - P_2A_2\sin\theta - F_y = \rho Q\,(V_2\cos\theta - 0)$$

$$F_y = \rho Q\,(-V_2\sin\theta) - P_2A_2\sin\theta$$

$$\text{Resultant Force } (F_R) = \sqrt{F_x^2 + F_y^2}$$

$$\boxed{\tan\theta = \frac{F_y}{F_x}}$$

Momentum Correction Factor

It is defined as the ratio of momentum of the flow per second based on the actual vel to the momentum of flow per second based on the average vel across the section. It is denoted by β

$$\beta = \frac{\text{momentum per second based on actual vel}}{\text{momentum per second based on avg vel}}$$