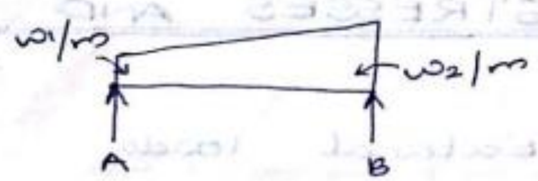
& w/m load at other end.

b) Trapezoidal loading



if the load at one end is w_1/m & load at other end w_2/m.

* Force → Anything which change or tends to change the state of a body.

* Types of force

Based on nature of action

1. Normal force

Force acting $\perp^r$ to the surface.

2. Shear force

Force acting tangential to the surface.

* Normal force

1. Tensile force

→ Force which causes increasing length of body

2. Compressive force.

→ Force which cause ↓ length of body

*** Stress : Force acting per Unit area**

$$S = \frac{F}{A}$$

$$\text{Stress} = \frac{\text{Force}}{A}$$

*** Strain :**

$$\text{Strain} = \frac{\text{change in dimension}}{\text{Original dimension}}$$

→ Types

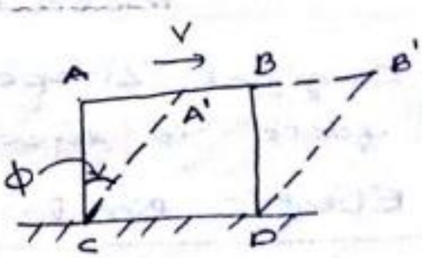
1. Linear Strain

$$LS = \frac{\text{change in length}}{\text{Original length}} = \frac{\delta l}{l}$$

2. Volumetric strain

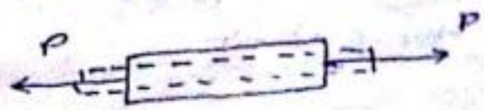
$$V.S = \frac{\delta V}{V}$$

3. Shear Strain



Angular displace-ment under the action of ~~other~~ shear stress.

$$\phi = \frac{AA'}{AC}$$



* **Stress :** Force acting per Unit area

$$S = \frac{F}{A}$$

$$Stress = \frac{Force}{A}$$

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→ **Types**

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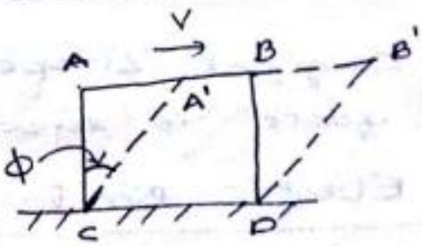
3. **Shear strain**



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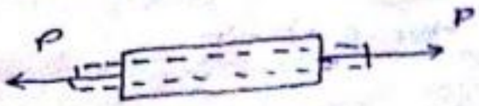
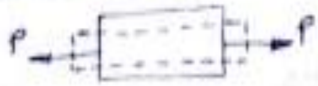


* Lateral strain :-



when a body is subjected to tensile stress its length increases but width decreases is called ~~decrease~~ lateral strain or the strain in the lateral dirⁿ or dirⁿ ⊥ʳ to the ~~length~~

* Poisson's Ratio

$$\frac{\text{Lateral strain}}{\text{Linear strain}} \left(\mu \text{ or } \frac{1}{m} \right)$$

* Elastic material

material which regains its original shape when the external force is removed.

* Elastic limit-

The limit up to which the material shows elastic property, the body will not regain its original shape if force is applied beyond the limit.

Hookes law :-

The stress is directly proportional to strain within its elastic limit

$$\frac{\text{stress}}{\text{strain}} = \text{Constant} = \text{modulus of elasticity}$$

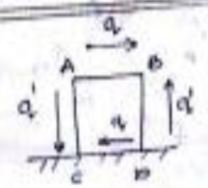
3 types of moduli

1. Young's modulus $= E = \dfrac{\text{Normal stress}}{\text{Linear strain}}$

2. Bulk modulus, $(K) = \dfrac{\text{Normal stress}}{\text{Volumetric strain}}$

3. Rigidity modulus $(G) = \dfrac{\text{Shear stress}}{\text{Shear strain}}$

* Complementary shear



Consider a block ABCD of unit width subject to shear stresses q shown in fig. The couple AB/CD formed by shear q shear stresses should be balanced by another couple q' acting on faces AC & BD. q' is called the complementary shear.

The clockwise moment due to 'q' acting on faces AB & CD up balanced by the anticlockwise moment due to q' acting on ~~~~ AC & BD

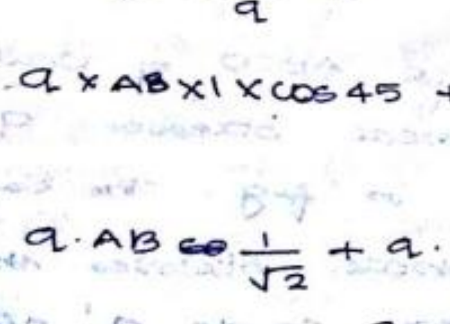
ie, $q \times AB \times 1 \times BD = q' \times AC \times 1 \times BD$

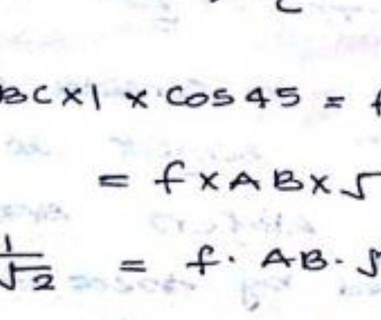
$$q = q'$$

* principle of complementary shear

The set of shear stress across a plane is always accompanied by a set of balancing shear stress of same intensity across the plane, and normal to it.

* Normal stress due to pure shear.





$$q \times AB \times 1 \times \cos 45 + q \times BC \times 1 \times \cos 45 = f \times AC \times 1$$

$$= f \times AB \times \sqrt{2}$$

$$q \cdot AB \cdot \frac{1}{\sqrt{2}} + q \cdot AB \cdot \frac{1}{\sqrt{2}} = f \cdot AB \cdot \sqrt{2}$$

$$\frac{q}{\sqrt{2}} + \frac{q}{\sqrt{2}} = f\sqrt{2}$$

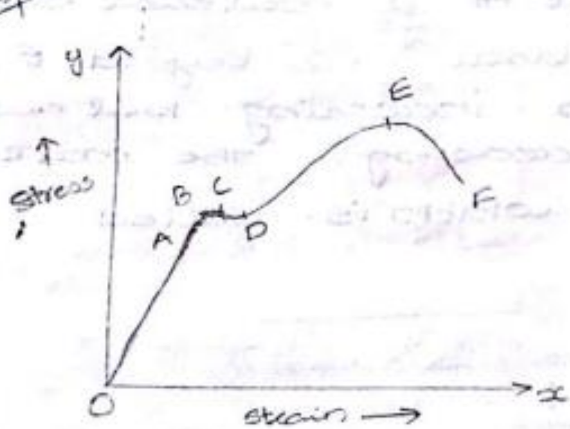
$$\frac{2q}{\sqrt{2}} = \sqrt{2}q = f\sqrt{2}$$

$$q = f$$

if a pair of 2 mutually ⊥r planes are subjected to shear stress normal stress of opposite will be produce 2 mutually ⊥r planes at an angle of 45° with them.

*) stress - strain diagram for mild steel.

imp.



O - origin
A - limit of proportionality.
B - Elastic limit
C - upper yield point (start permanent deformation)
D - Lower
E - ultimate stress
F - Breaking point.

The figure shows the stress-strain curve for tension-stress on mild steel. The ~~ten~~ curve starts from origin 'O' since the stress is zero (offset strain). The point A shows the limit of proportionality. ie, the portion 'OA' Hookes law is obeyed. 'B' is the elastic limit of is small 'AB' we get a curve which is the special property for mild steel. 'C' is the upper yield

* Elongation of a Uniform Bar.

Let, l – length of bar

A – Area of cross section

E – youngs modulus

δl – Elongation

P – Applied load.

$$E = \frac{\text{stress}}{\text{strain}}$$

$$= \frac{\frac{P}{A}}{\frac{\delta l}{l}}$$

$$\frac{E.\delta l}{l} = \frac{P}{A}$$

$$\delta l = \frac{Pl}{EA}$$

* A steel rod 50 cm long & 20 mm dia is subjected to axial tensile force of 20 kN. Find the elongation of rod if $E = 200$ GPa   GPa $= 10^9$ N/m²

Ans)

$$\delta l = \frac{Pl}{EA}$$

$l =$ 50 cm $=$ 5000 mm $= 0.5$ m

$D =$ 20 cm $= 0.02$ m

$A = \pi r^2 = \frac{\pi D^2}{4} = \cancel{\frac{\pi \cdot 20^2}{4}}$

$= \cancel{\frac{\pi}{4} \cdot 400 = 100\pi}$

$= \frac{\pi}{4}(0.02)^2 = \pi \times 10^{-4}$

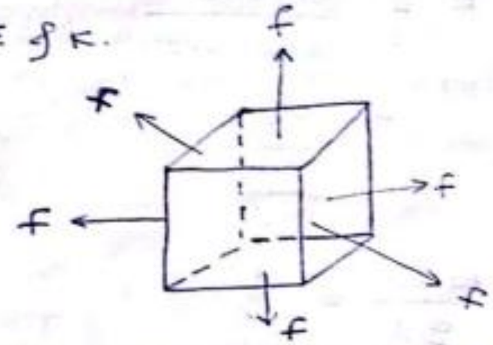
$$= \frac{20\times10^3 \times 0.5}{200\times10^9 \times \pi\times10^{-4}}$$

$$= \frac{0.5}{10^3 \times \pi} = \underline{\underline{1.59 \times 10^{-4}\ m}}$$

* Relationship b/w Elastic Constants.

E - youngs modulus
K - Bulk modulus
G - Rigidity
$\frac{1}{m}$ - poisson's ratio

→ Relationship b/w E & K.



consider a Cube ~~of~~ with unit side dir^n subjected to normal stresses 'f' in 3 mutually ⊥^r dir^n. ie, x, y & z dir^n as shown in figure.

$E = \frac{\text{Stress}}{\text{Strain}}$

Strain in x dir^n, $e_x = \cancel{\frac{f}{E}} \cancel{+ \frac{\ }{m}} \cdot \frac{f}{E}$

$$e_x = \frac{f}{E} - \frac{1}{m} \cdot \frac{f}{E} - \frac{1}{m} \cdot \frac{f}{E}$$

poissons ratio = $\frac{\text{lateral strain}}{\text{linear strain}}$

$$e_x = \frac{f}{E}\left(1 - \frac{2}{m}\right)$$

Similarly

$$e_y = e_z = \frac{f}{E}\left(1 - \frac{2}{m}\right)$$

Volumetric Strain, $e_v = \frac{\delta V}{V}$

$$e_v = \frac{(1+e_x)(1+e_y)(1+e_z) - 1}{1}$$

Original Volume $= 1$ (Cube is uni

=

$$e_v = 1 + e_x + e_y + e_z + (e_x e_y + e_y e_z + e_z e_x + e_x e_y e_z) - 1$$

≈ 0

neglecting $e_z \approx 0$, $e_x e_y \approx 0$

$$e_v = e_x + e_y + e_z = \frac{\delta V}{V}$$

$$\frac{\delta V}{V} = \frac{3f}{E}\left(1 - \frac{2}{m}\right)$$

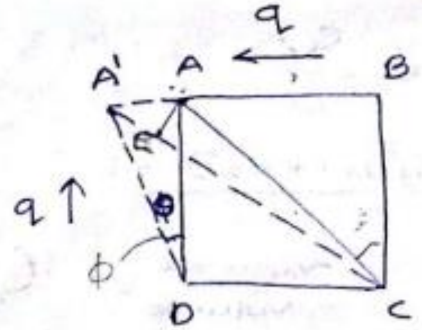
$$K = \frac{f}{e_v} = \frac{f}{\frac{3f}{E}\left(1 - \frac{2}{m}\right)}$$

$$K = \frac{E}{3\left(1 - \frac{2}{m}\right)}$$

$$\boxed{E = 3K\left(1 - \frac{2}{m}\right)}$$

* Relationship b/w E & G

$$E = 3K\left(1 - \frac{2}{m}\right)$$



Consider a Cube of unit side dimension subjected to pure shear stress 'q' as shown figure. let ϕ be the angular displacement (shear strain) & let point 'A' be shifted to A' due to the applied shear stress.

Linear strain in diagonal AC,

$$= \frac{q}{E} + \frac{1}{m} \cdot \frac{q}{E}$$

$$= \frac{q}{E}\left(1 + \frac{1}{m}\right) \quad \text{— ①}$$

Strain in AC $= \dfrac{A'E}{CE} = \dfrac{\text{change in diagonal dimension}}{\text{Original}}$

$$= \frac{AA' \cos 45}{\left(\frac{AD}{\cos 45}\right)}$$

$\dfrac{AA'}{AD} = \sin\phi$

$\sin\phi \sim 0$

$\sin\phi \approx \phi$

$$= \frac{AA'\left(\frac{1}{\sqrt{2}}\right)}{\frac{AD}{\left(\frac{1}{\sqrt{2}}\right)}} = \frac{AA'}{2AD}$$

$$= \frac{\phi}{2} = \frac{q}{2G} \quad \text{— ②}$$

① & ② ⇒ $\frac{q}{E}\left(1+\frac{1}{m}\right) = \frac{q}{2G}$

$$E = 2G\left(1+\frac{1}{m}\right)$$

*) Relationship b/w E, G & K.

$$E = 2G\left(1+\frac{1}{m}\right) = 3K\left(1-\frac{2}{m}\right)$$

$$2G + \frac{2G}{m} = 3K - \frac{6K}{m}$$

$$2G - 3K = -\frac{2G}{m} - \frac{6K}{m} = \frac{-1}{m}(2G+6K)$$

$$\frac{1}{m} = \frac{2G-3K}{-(2G+6K)}$$

$$\frac{1}{m} = \frac{3K-2G}{2G+6K}$$

$$E = 2G\left(1+\frac{1}{m}\right) = 2G\left(1+\frac{3K-2G}{2G+6K}\right)$$

$$E = 2G\left(\frac{2G+6K+3K-2G}{2G+6K}\right)$$

$$= 2G\left(\frac{9K}{2G+6K}\right) = \frac{18GK}{2G+6K}$$

$$\boxed{E = \frac{9GK}{G+3K}}$$

* A bar of Certain material of size 60mm x 60 mm is subjected to an axial pull of 200 KN. The extension observed over a gage length of 150 mm is 0.1 mm & decrease in each side is 0.008 mm. Determine the elastic moduli of the material of the bar.

Ans) $E = \frac{9KG}{3K+G}$

$E = \frac{\text{Stress}}{\text{Strain}}$

$\text{Stress} = \frac{P}{A} = \frac{200 \times 10^3}{60 \times 60 \times 10^{-6}} = \frac{2 \times 10^3}{36}$

$= \frac{10^3}{18} = 0.05 \times 10^9$

$= \underline{\underline{5 \times 10^7 \, N/m^2}} = 55.5 \, N/mm^2$

$\text{Strain} = \frac{\Delta l}{l} = \frac{0.1}{150} = 6.67 \times 10^{-4}$

$E = \frac{55.5}{6.67 \times 10^{-4}} = 8.33 \times 10^4 \, N/mm^2$

$E = 2G\left(1 + \frac{1}{m}\right)$

$\frac{1}{m} = \frac{0.008}{60} = \text{lateral strain} = 1.33 \times 10^{-4}$

linear strain $= \frac{0.1}{150} = 6.67 \times 10^{-4}$

poissons ratio, $= \frac{1}{m} = \frac{\text{lateral}}{\text{linear}} = \frac{1.33 \times 10^{-4}}{6.67 \times 10^{-4}}$

$= 0.2$

$$E = 2G\left(1+\frac{1}{m}\right) = 2G(1+0.2) = 2.4\,G$$

$$G = \frac{E}{2.4} = \frac{8.33\times10^4}{2.4} = 3.47\times10^4\ N/mm^2$$

$$E = 3K\left(1-\frac{2}{m}\right) = 3K(1-2\times0.2)$$

$$= 3K(1-0.4) = 3K\times0.6 = 1.8\times K$$

$$K = \frac{E}{1.8} = \frac{8.33\times10^4}{1.8} = 4.62\times10^4\ N/mm^2$$

*) A bar of ~~200~~ 20 mm dia is tested in tension it is observed that when a load of 37.7 KN is applied the extension measured over a gage length of 200 mm is 0.12 mm. and the contraction is dia is 0.0036 mm. Find poissons ratio and elastic moduli.

Ans) $E = \frac{stress}{strain}$

$$= \frac{\frac{P}{A}}{\frac{\delta l}{l}} = \frac{\frac{P}{\frac{\pi D^2}{4}}}{\frac{\delta l}{l}} = \frac{\frac{37.7\times10^3}{\frac{\pi}{4}\times20^2}}{\frac{0.12}{200}}$$

$$= \frac{120.002}{6\times10^{-4}} = 20\times10^4$$

Linear strain, $= \frac{\delta l}{l} = \frac{0.12}{200}$

$$= 6\times10^{-4}$$