MODULE - III

MOMENT DISTRIBUTION METHOD.

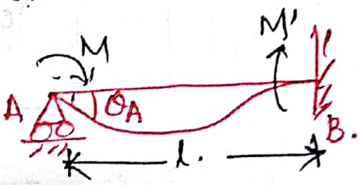
Moment Distribution Method also known as Hardy Cross Method is a displacement method of analysis which provides a convenient method of analysis of indeterminate beams & frames. This is basically an iterative process. The procedure in general involves artificially restraining temporarily all the joints against rotation & writing down the fixed end moments for all the members. At this point, there will be unbalanced moments. Now the joints are released one by one in succession. At each release joints, there will be rotation till the moments are balanced.

[The balancing moments are distributed to all the ends of the members meeting at that joint. A certain fraction of these moments are then carried over to the far end of the member]

TERMINOLOGIES

(1) CARRY OVER MOMENT :

When a moment is applied, at one end of a member, allowing rotation at that end & fixing the far end, some moment develops at the far end also. This moment is known as carry over moment. Eg: In a beam AB, as shown in the fig. if M is the applied moment at end A allowing rotation of A, then a moment is developed at B (M'). This M' moment is called as carry over moment. The ratio of carry over moment to the applied moment is called as carry over Factor.



$$\text{carry over factor} = \frac{M'}{M}$$

$$= \frac{\text{carry over moment}}{\text{applied moment}}$$

(ii) Stiffness

Moment Required to rotate an end by unit angle when rotation is permitted at that end is called as stiffness of the beam. In the above fig, if θ_A is the rotation @ end A. Then stiffness of the beam,

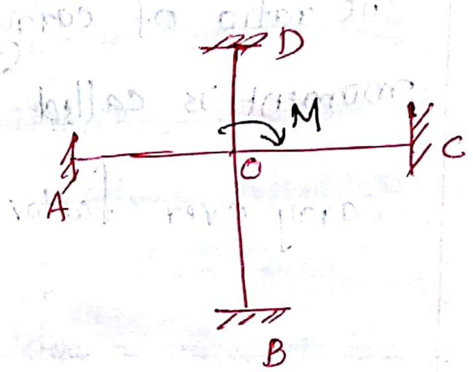
$$\boxed{K = \frac{M}{\theta_A}}$$

(iii) Distribution factor & Distribution moment.

When a moment is applied to a rigid joint where a no. of members are meeting, the applied moment is shared by the members meeting at the joint. The moment shared by each member is called as distribution moment.

The ratio of distribution moment to the applied moment at a joint is called as Distribution factor of that member. Thus, if M_{OA} is the moment shared by member OA when a moment, M is applied at joint O, then the Distribution factor for member OA is given by:

$$\boxed{d_{OA} = \frac{M_{OA}}{M}}$$

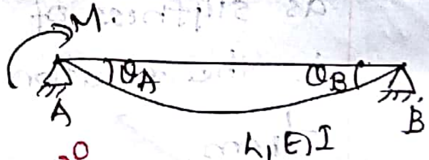


* Follow the sign conventions same as that in slope-deflection method.

FUNDAMENTAL PROPOSITIONS:

(i) Beams hinged at both the ends:-

Slope deflection Eqn:-



$$M_{AB} = \cancel{M_{FAB}}^{\,0} + \frac{2EI}{l}\left[2\theta_A + \theta_B - \cancel{\frac{3\Delta}{L}}^{\,0}\right]$$

$$M_{BA} = \cancel{M_{FBA}}^{\,0} + \frac{2EI}{l}\left[\theta_A + 2\theta_B - \cancel{\frac{3\Delta}{L}}^{\,0}\right]$$

$$\therefore M_{AB} = \frac{2EI}{L}\left[2\theta_A + \theta_B\right] = M \text{ — (1)}$$ {∵ Moment M acting @ A}

$$M_{BA} = \frac{2EI}{L}\left[\theta_A + 2\theta_B\right] = 0 \text{ — (2)}$$ ∵ hinge support

(2) ⇒ $\frac{2EI}{L}\left[\theta_A + 2\theta_B\right] = 0$

$$\theta_A = -2\theta_B$$

Substituting in eqn ①,

$$M = \frac{2EI}{L}\left[2\theta_A - \frac{1}{2}\theta_A\right] \quad \text{or} \quad M = \frac{2EI}{L}\left[-4\theta_B + \theta_B\right]$$

$$= \frac{2EI}{L} \times \frac{3}{2}\theta_A \qquad\qquad = -\frac{6EI}{L}\theta_B$$

$$\boxed{M = \frac{3EI}{L}\theta_A.}$$

The rotational stiffness for a member with far end hinged is given by:

$$M = K_{AB} = \frac{3EI}{L}$$

{Rotational stiffness is the moment required for unit rotation, ie, substitute $\theta_A = 1$}

Thus we can say the moment K required to rotate the near end of a prismatic beam through unit angle without translation the far end being hinged is given by,

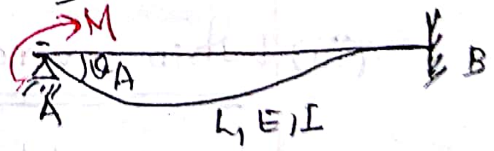
$$\boxed{K = \frac{3EI}{L}}$$

(ii) Beam with far end Fixed :-

Slope-deflection Eqn:

$$M_{AB} = \cancel{M_{FAB}}^{0} + \frac{2EI}{L}\left[2\theta_A + \cancel{\theta_B}^{0} - \cancel{\frac{3\Delta}{L}}^{0}\right]$$

$$M_{BA} = \cancel{M_{FBA}}^{0} + \frac{2EI}{L}\left[2\cancel{\theta_B}^{0} + \theta_A - \cancel{\frac{3\Delta}{L}}^{0}\right]$$



$$M_{AB} = \frac{2EI}{L} \times 2\theta_A = M \quad \text{— ①}$$

$$\boxed{M_{AB} = \frac{4EI}{L}\theta_A}$$

The rotational stiffness for a member when the far end is hinged ($\theta_A = 1$) is given by;

$$\boxed{M = K_{AB} = \frac{4EI}{L}}$$

$$M_{BA} = \cancel{M_{FBA}}^{0} + \frac{2EI}{L}\left[\theta_A + \cancel{2\theta_B}^{0} - \cancel{\frac{3\Delta}{L}}^{0}\right].$$

$$\boxed{M_{BA} = \frac{2EI}{L}\theta_A = \frac{M}{2}}$$

Thus we can say that the moment 'K' required to rotate the near end of a prismatic beam without translation, far end being fixed is given by,

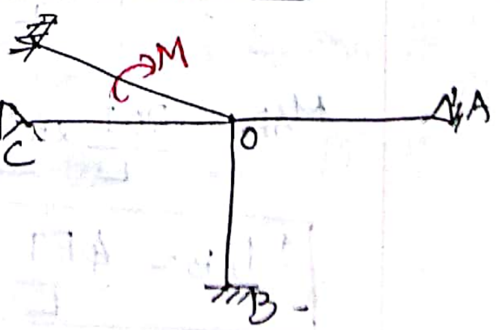
$$\boxed{K = \frac{4EI}{L}}$$

Also, a moment which rotates the near end of a prismatic beam without translation, the far end being fixed induces at the far end, a moment of half its magnitude & of same direction.

(iii) When several members meet at a joint

$$K = \frac{M}{\theta}.$$

$$M = K\theta.$$



$$M_{OA} = K_{OA}\,\theta = \frac{3EI}{L} \times \theta.$$

$$M_{OB} = K_{OB}\cdot\theta = \frac{4EI}{L} \times \theta.$$

$M_{OC} = K_{OC} \times \theta = \frac{3EI}{L} \times \theta$

$M_{OD} = K_{OD} \times \theta = \frac{4EI}{L} \times \theta$

$M = M_{OA} + M_{OB} + M_{OC} + M_{OD}$.

$\therefore$ Distribution factor, $DF = \frac{M_{OA}}{M} = \frac{K_{OA} \times \theta}{K_{OA}\times\theta + K_{OB}\times\theta + K_{OC}\theta + K_{OD}\theta}$

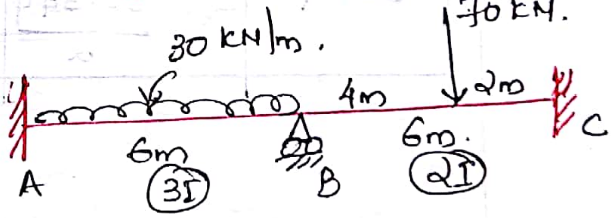
$$\therefore \boxed{DF = \frac{K_{OA}}{\Sigma K}}$$

Distribution moment, $DM = M_{OA} = \frac{K_{OA}}{\Sigma K} \times M$.

Thus we can say a moment which tends to rotate a joint without translation, will be divided among the connecting members at the joint in the proportion of their stiffnesses $\left\{\frac{k_1}{\Sigma k}, \frac{k_2}{\Sigma k}, \frac{k_3}{\Sigma k}, \ldots\right\}$

This ratio $\frac{k}{\Sigma k}$ is called as the distribution factor.

Q. Analyse the continuous beam by moment-distribution method.



Soln. Fixed End moments:

$M_{FAB} = -\frac{wl^2}{12} = -\frac{30\times 6^2}{12} = \underline{-90 \text{ kNm}}$.

$M_{FBA} = \frac{wl^2}{12} = \underline{90 \text{ kNm}}$

$M_{FBC} = -\frac{Wab^2}{l^2} = -\frac{70\times 4\times 2^2}{6^2} = \underline{-31.11 \text{ kNm}}$.

$M_{FCB} = \frac{Wa^2b}{l^2} = \frac{70\times 4^2\times 2}{6^2} = \underline{62.22 \text{ kNm}}$.

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Calculation of Distribution Factor, DF.

Joint	Member	k	ΣK	$DF = \frac{k}{\Sigma K}$
B	BA	$\because$ Far end fixed. $\frac{4E \times 3I}{L} = 2EI$	$\frac{10}{3}EI$	0.6
	BC	$\frac{4E[2I]}{L} = \frac{4}{3}EI$		0.4

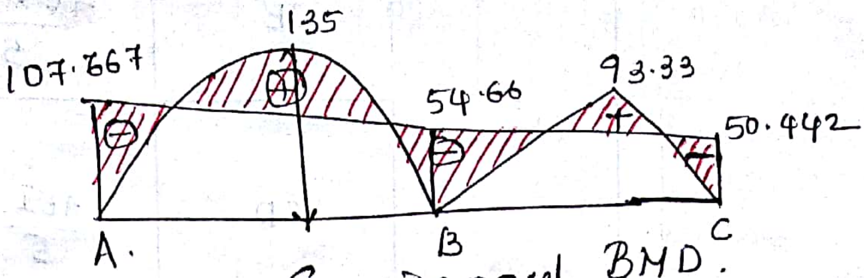
Joint	A	B		C
Member	AB	BA	BC	CB
DF		0.6	0.4	
FEM	−90	Unbalanced $M = 90 - 31.11 = 58.89$ $\boxed{90}$	$\boxed{-31.11}$ $\therefore$ Balanced $M = -58.89$	62.22
DM	0	−35.344 0.6×-58.89	−23.556 0.4×-58.89	0
COM	−17.667 $\frac{-35.344}{2}$	0	0	−11.778 $\frac{-23.556}{2}$

[COM at fixed far end $- \frac{M}{2}$]

Since there is no further moment left to distribute @ joint B we can stop the analysis.

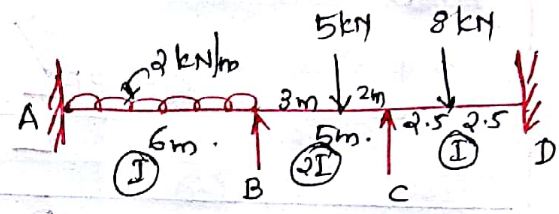
FINAL MOMENT

−90−17.667	90−35.344	−31.11−23.556	62.22−11.778
−107.667	54.656	−54.656	50.442



Superimposed BMD.

Q. Analyse the continuous beam shown in the fig. using moment distribution method.



Soln. Fixed End Moments:

$$M_{FAB} = -\frac{wl^2}{12} = -\frac{2\times6^2}{12} = \underline{\underline{-6\ kNm.}}$$

$$M_{FBA} = \frac{wl^2}{12} = \underline{\underline{6\ kNm.}}$$

$$M_{FBC} = -\frac{Wab^2}{l^2} = -\frac{5\times3\times2^2}{5^2} = \underline{\underline{-2.4}}\ kNm.$$

$$M_{FCB} = \frac{Wa^2b}{l^2} = \frac{5\times3^2\times2}{5^2} = \underline{\underline{3.6\ kNm.}}$$

$$M_{FCD} = -\frac{Wl}{8} = \frac{-8\times5}{8} = \underline{\underline{-5}}\ kNm.$$

$$M_{FDC} = \frac{Wl}{8} = \underline{\underline{5\ kNm}}.$$

Calculation of Distribution factor (DF)

Joint	Member	k	Σk	$DF = \frac{k}{\Sigma k}$.
B	BA	$\frac{4EI}{6}$	$\frac{34}{15}EI$	0.294
	BC	$\frac{4E(2I)}{5}$		0.71

While analysing B, consider all other joints fixed.

C	CB	$\frac{4E\times 2I}{5}$	$\frac{12}{5}EI$	0.67
	CD	$\frac{4EI}{5}$		0.33

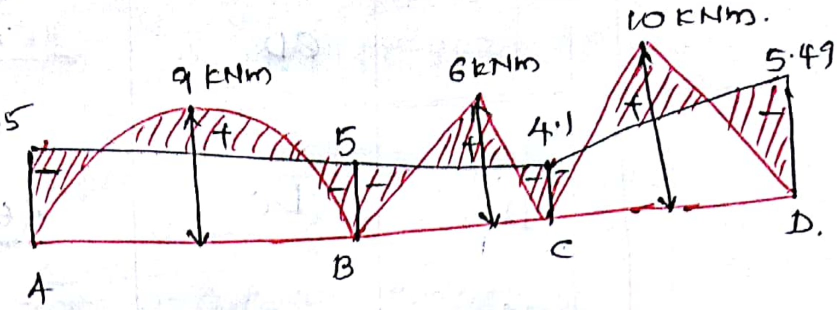
Note:- If the end support is hinged or overhang, then we will use :

$$K = \frac{3EI}{L}$$

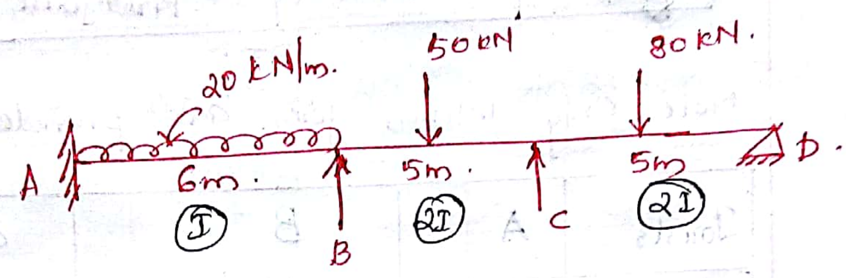
Joints	A	B		C		D
Members	AB	BA	BC	CB	CD	DC
DF		0.29	0.71	0.67	0.33	
FEM	−6	UN BAL MOMENT = 3.6 6	−2.4 BAL M = −3.6	UN BAL MOMENT = −1.4 3.6	−5 BAL MOM = 1.4	5
DM	0	−1.044 0.29 × −3.6	−2.556 0.71 × −3.6	0.938 0.67 × 1.4	−0.462 0.33 × 1.4	0
COM	−0.522 $\frac{-1.044}{2}$	0	0.469 $\frac{0.938}{2}$	−1.278 $\frac{-2.556}{2}$	0	0.231 $\frac{0.462}{2}$
DM	0	0.136	−0.333	0.856	0.421	0
COM	0.068	0	0.428	0.1665	0	0.211
DM	0	−0.124	−0.304	−0.11	−0.054	0
COM	−0.062	0	−0.055	−0.152	0	0.027
DM	0	0.015	0.039	0.101	0.0501	0
COM	7.5×10^{-3}	0	0.05	0.019	0	0.025
DM	0	−0.0145	−0.035	−0.012	-6.27×10^{-3}	0
COM	-7.25×10^{-3}	0	-6×10^{-3}	−0.0175	0	-3.135×10^{-3}

FINAL Moment	−6.516	4.9685	−5.12	4.096	−4.12	5.49.

Superimposed BMD.



Q.



Soln. Fixed end moments.

$$M_{FAB} = -\frac{wl^2}{12} = -\frac{20\times 6^2}{12} = \underline{\underline{-60}}\ kNm.$$

$$M_{FBA} = \frac{wl^2}{12} = \underline{60\ kNm}.$$

$$M_{FBC} = -\frac{Wl}{8} = -\frac{50\times 5}{8} = \underline{\underline{-31.25}}\ kNm.$$

$$M_{FCB} = \frac{Wl}{8} = \underline{\underline{31.25}}\ kNm.$$

$$M_{FCD} = -\frac{Wl}{8} = -\frac{80\times 5}{8} = \underline{\underline{-50}}\ kNm.$$

$$M_{FDC} = \frac{Wl}{8} = \underline{\underline{50\ kNm}}.$$

Calculation of Distribution factor (DF):

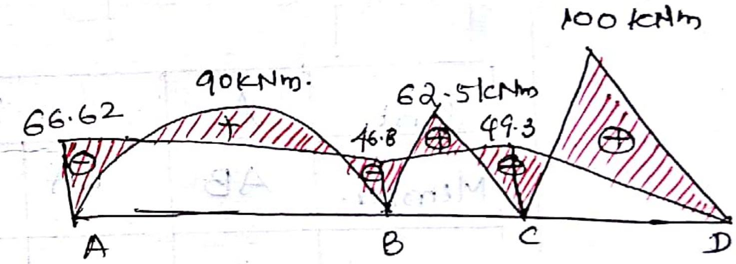
Joint	Member	k	Σk	$DF = \frac{k}{\Sigma F}$
B	BA	$\frac{4EI}{6}$	$\frac{34}{15}EI$	0.29
	BC	$\frac{4E\times 2I}{5}$		0.71

C	CB	$\frac{4E\times 2I}{5}$	$\frac{14}{5}EI$	0.57
	CD	D hinged joint. $\frac{3E\times 2I}{5}$		0.43.
D	DC	$\frac{4E\times 2I}{5}$ Fixed joint	$\frac{4E\times 2I}{5}$	1

Note: Only internal joints are provided with fixity.

Joints	A	B		C		D.
Member	AB	BA	BC	CB	CD	DC
DF		0.29	0.71	0.57	0.43	1
FEM	−60	60	−31.25	31.25.	−50	BAL M= −50 50
Balancing						−50×1 −50
COM					−50/2 −25	0
Add Final M @ cond. I	−60	BAL M = 28.75 $\boxed{60}$	$\boxed{-31.25}$	BAL M = 43.75 $\boxed{31.25}$	$\boxed{-75}$ −50 −25	0
DM	0	−8.33	−20.41	24.937	18.812	
COM	−4.165	BAL M = −12.47 $\boxed{0}$	$\boxed{12.47}$	BAL M = 10.21 $\boxed{-10.21}$	$\boxed{0}$	
DM	0	−3.62	−8.85	5.81	4.39	
COM	−1.81	BAL M = −2.91 $\boxed{0}$	$\boxed{2.91}$	BAL M = 4.425 $\boxed{-4.425}$	$\boxed{0}$	
DM	0	−0.84	−2.1	2.52	1.90	

		BAL M = -1.26		BAL M = 1.05		
COM	-0.42	0	1.26	-1.05	0	
DM	0	-0.365	-0.894	0.59	0.45	
		BAL M = -0.295		BAL M = 0.447		
COM	-0.182	0	0.295	-0.447	0	
DM	0	-0.085	-0.209	0.254	0.192	
COM	-0.0425	0	-0.127	0.1045	0	
FINAL MOMENT	-66.62	46.76	-49.4	49.93	-49.25	0



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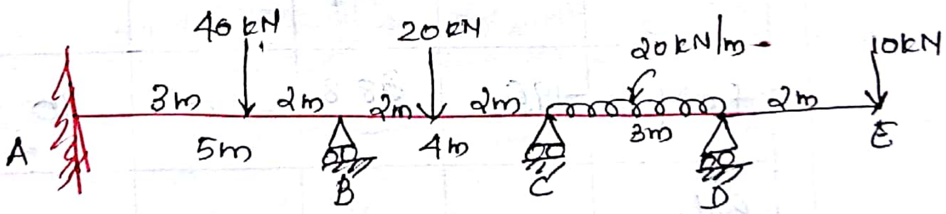

Soln. Fixed End Moments:

$$M_{FAB} = -\frac{Wab^2}{l^2} = \frac{-40 \times 3 \times 2^2}{5^2} = \underline{\underline{-19.2 \text{ kNm}}}.$$

$$M_{FBA} = \frac{Wa^2b}{l^2} = \frac{40 \times 3^2 \times 2}{5^2} = \underline{\underline{28.8 \text{ kNm}}}.$$

$$M_{FBC} = \frac{-Wl}{8} = \frac{-20 \times 4}{8} = \underline{\underline{-10 \text{ kNm}}}$$

$$M_{FCB} = \frac{Wl}{8} = \underline{\underline{10 \text{ kNm}}}$$

$$M_{FCD} = -\frac{Wl^2}{12} = \frac{-20 \times 3^2}{12} = \underline{\underline{-15 \text{ kNm}}}.$$

$$M_{FDC} = \frac{wl^2}{12} = \underline{\underline{15 \text{ kNm}}}.$$ $$M_{FDE} = \underline{\underline{-20 \text{ kNm}}}.$$

Calculation of Distribution Factor DF :

Joint	Member	k	Σk	$DF = \frac{k}{\Sigma k}$
A	AB			0 Fixed support
B	BA	$\frac{4EI}{5}$	$\frac{9}{5}EI$	0.44
	BC	$\frac{4EI}{4}$		0.56
C	CB	$\frac{4EI}{4}$	$2EI$	0.5
	CD	$\frac{3EI}{3}$		0.5
D	DC	$\frac{4}{3}EI$	$\frac{4}{3}EI$	1

Joint	A	B		C		D	
Member.	AB	BA	BC	CB	CD	DC	DE
DF	0	0.44	0.56	0.5	0.5	1	0
FEM	-19.2	28.8	-10	10	-15	BAL M = -5 15	-20
Bal.						5×1 5	5×0 0
COM					5/2 2.5		
Final M.	-19.2	28.8	-10	10	-12.5	20	-20
DM	0	-8.27	-10.53	1.25	1.25	0	-
COM	-4.14	0	0.63	5.27	0	-	-
	-23.34	20.53	-19.9	5.98	-11.25	20	-20

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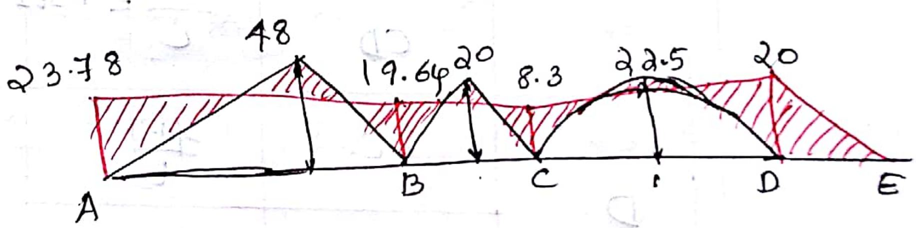
DM	0	−0.28	−0.35	2.64	2.64	0	
COM	−0.14	0	1.32	−0.175	0		
	−23.48	20.25	−18.93	8.445	−8.61	20	−20
DM	0	+0.58	−0.739	+0.08	+0.08	0	
COM	−0.29	0	+0.04	−0.34	0		
	−23.77	19.67	−19.62	8.155	−8.53	20	−20
DM	0	−0.022	−0.049	0.165	+0.187	0	
COM	−0.011	0	0.0825	−0.014	0		
Final Moments	−23.78	19.64	−19.6	8.3	−8.3	20	−20



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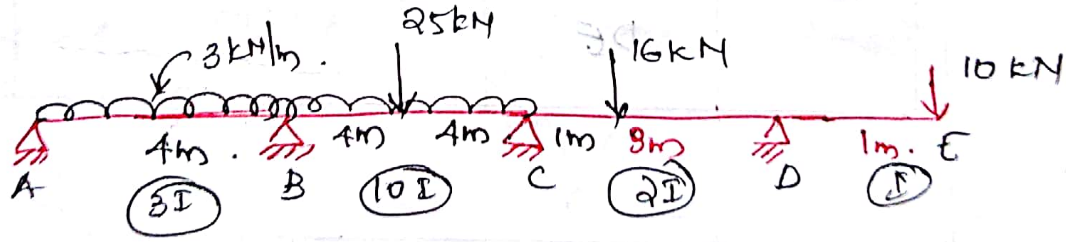


Soln Fixed End Moments:

$$M_{FAB} = \frac{-wl^2}{12} = \frac{-3\times 4^2}{12} = -4 \text{ kNm}$$

$$M_{FBA} = \frac{wl^2}{12} = 4 \text{ kNm}.$$

$$M_{FBC} = -\frac{wl^2}{12} + -\frac{Wl}{8} = -\frac{3\times 8^2}{12} + -\frac{25\times 8}{8} = -41 \text{ kNm}$$

$$M_{FCB} = \frac{wl^2}{12} + \frac{Wl}{8} = \frac{3\times8^2}{12} + \frac{25\times8}{8} = 41\ kNm.$$

$$M_{FCD} = \frac{-Wab^2}{l^2} = -\frac{16\times1\times3^2}{4^2} = -9\ kNm.$$

$$M_{FDC} = \frac{Wa^2b}{l^2} = \frac{16\times1^2\times3}{4^2} = 3\ kNm.$$

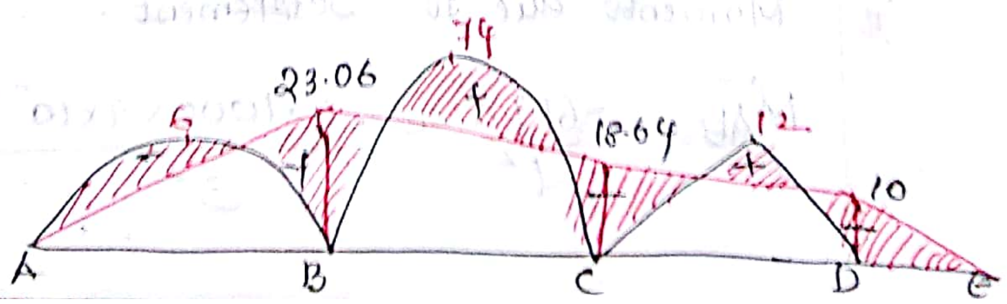
$$M_{FDE} = -10\ kNm.$$

Calculation of Distribution factor DF :

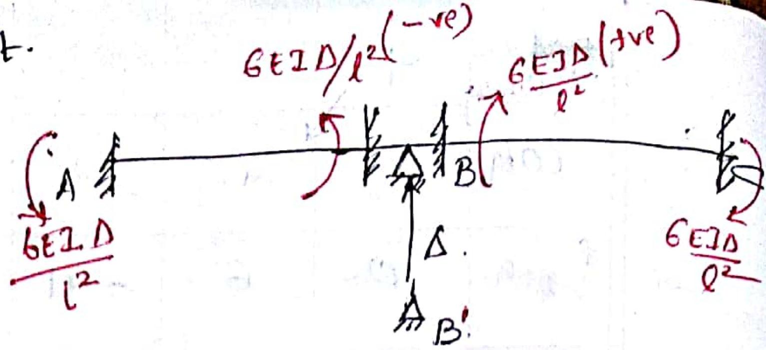
Joint	Member	k	Σk	$DF = \frac{k}{\Sigma F}$
B	BA	$\frac{3EI}{l} = \frac{9EI}{4}$	7.25 EI	0.31
	BC	$\frac{4EI}{l} = 5EI$		0.68
C	CB	$\frac{4EI}{L} = 5EI$	6.5EI	0.76
	CD	$\frac{3EI}{L} = 1.5EI$		0.23
D.	DC	$\frac{4EI}{l}$	$\frac{4EI}{L}$	1
	DE			0

Joint	A	B		C		D.	
Member	AB	BA	BC	CB	CD	DC	DE
DF	0	0.31	0.689	0.76	0.23	1	0
FEM	-4	4	-41	41	-9	3	-10

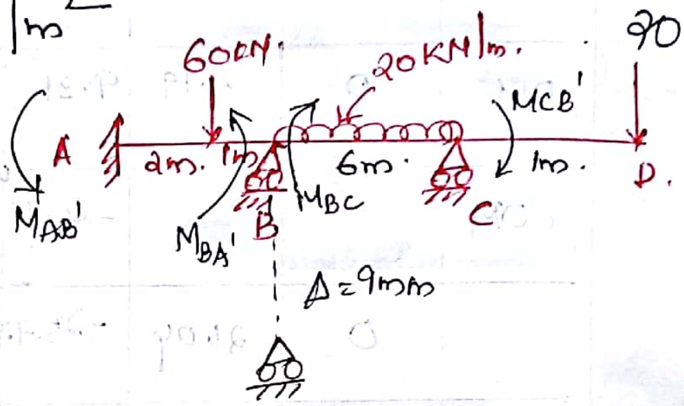
~~DM~~ Balancing	4					7	0
COM		2			3.5		
FEM	0	6	−41	41	−55	10	−10
DM	0	10.85	24.115	−26.98	−8.165		
COM			−13.49	12.05			
	0	16.85	−30.375	26.07	−13.665	10	−10
DM	0	4.19	9.31	−9.42	−2.85		
COM			−4.71	4.65			
	0	21.04	−25.77	21.3	−16.515	10	−10
DM	0	1.46	3.25	−3.63	−1.10		
COM			−1.82	1.62			
	0	22.5	−24.34	19.29	−17.615	10	−10
DM	0	0.5704	1.267	−1.28	−0.32		
			−0.64	0.63			
Final moment	0	23.06	−23.7	18.64	−18	10	−10



Sinking of Support.



Q. Analyse the continuous beam shown in fig. using Moment Distribution method & draw the BMD. Support B settles by 9mm. Take $EI = 1000 kN/m^2$



Soln. I. Fixed End Moments :-

$$M_{FAB} = -\frac{Wab^2}{l^2} = \frac{-60\times 2\times 1^2}{3^2} = \underline{-13.33\ kNm.}$$

$$M_{FBA} = \frac{Wa^2b}{l^2} = \frac{60\times 2^2\times 1}{3^2} = \underline{26.67\ kNm}$$

$$M_{FBC} = -\frac{wl^2}{12} = -\frac{20\times 6^2}{12} = \underline{-60\ kNm.}$$

$$M_{FCB} = \frac{wl^2}{12} = \underline{60\ kNm.}$$

$$M_{FCD} = -20\times 1 = \underline{-20\ kNm.}$$

II Moments due to settlement.

$$M'_{AB} = \frac{-6EI\Delta}{l^2} = \frac{-6\times 1000\times 9\times 10^{-3}}{3^2} = \underline{-6\ kNm.}$$

$$M_{BA}' = -\frac{6EI\Delta}{l^2} = -\frac{6\times1000\times9\times10^{-3}}{3^2} = \underline{-6 \text{ kNm}}.$$

$$M_{BC}' = \frac{6EI\Delta}{l^2} = \frac{6\times1000\times9\times10^{-3}}{6^2} = 1.5 \text{ kNm}.$$

$$M_{CB}' = \frac{6EI\Delta}{l^2} = \underline{1.5 \text{ kNm}}.$$

III Net Moment :

$$M_{AB}'' = -13.33 - 6 = -19.33 \text{ kNm}.$$

$$M_{BA}'' = 26.67 - 6 = 20.67 \text{ kNm}.$$

$$M_{BC}'' = -60 + 1.5 = -58.5 \text{ kNm}.$$

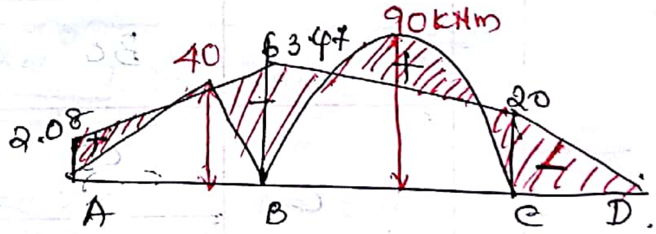
$$M_{CB}'' = 60 + 1.5 = 61.5 \text{ kNm}$$

IV Calculation of DF.

Joint	Member	k	Σk	$DF = \frac{k}{\Sigma F}$
B	BA	$\frac{4EI}{3}$	$\frac{11}{6}EI$	0.73
	BC	$\frac{3EI}{6}$		0.27
C	CB	$\frac{4EI}{6}$	$\frac{4}{6}EI$	1
	CD	0		0

V Moment Distribution Method:-

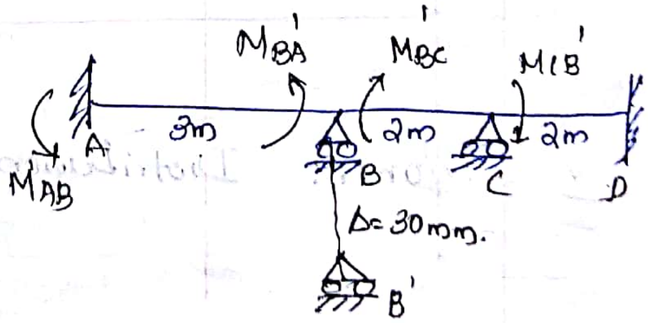
Joint	A	B		C	
Member.	AB	BA	BC	CB	CD
DF	—	0.73	0.27	1	0
FEM	−19.33	20.66	−58.55	BAL M = −41.5 $\boxed{61.5}$	$\boxed{-20}$
Bal.				−41.5 −41.5×1	
COM			−20.75		
ΣM.	−19.33	BAL M = 58.64 $\boxed{20.66}$	$\boxed{-79.3}$	20	−20
DM	0	42.81	15.83		
COM	21.41	0	0		
Final moment	2.08	63.47	−63.47	20	−20



Q. B Sinks by 30 mm

$E = 2 \times 10^5$ N/mm²

$I = 3 \times 10^6$ mm^4



Solⁿ I. No load, ∴ No FEM.

II) Moments due to settlement.

$$M'_{AB} = -\frac{6EI\Delta}{l^2} = -\frac{6\times 2\times10^5 \times 3\times10^6 \times 30}{3000^2}$$

$$= -12\times10^6 \text{ Nmm} = -12 \text{ kNm}$$

$$M'_{BA} = -\frac{6EI\Delta}{l^2} = -12 \text{ kNm}.$$

$$M'_{BC} = \frac{6EI\Delta}{l^2} = \frac{6\times 2\times 10^5\times 3\times10^6\times 30}{2000^2}$$

$$= 27\times10^6 \text{ Nmm} = 27 \text{ kNm}.$$

$$M'_{CB} = \frac{6EI\Delta}{l^2} = 27 \text{ kNm}$$

III) Net moments :

$M''_{AB} = -12$ kNm, $M''_{BA} = -12$ kNm,

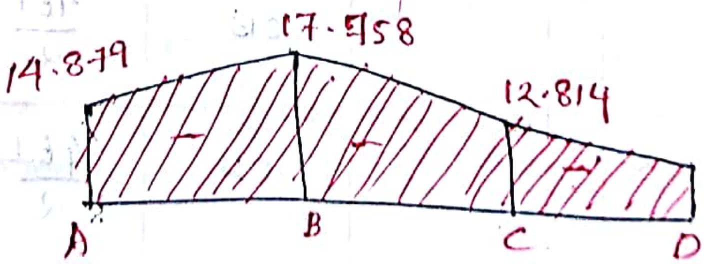
$M''_{BC} = 27$ kNm, $M''_{CB} = 27$ kNm, $M''_{CD} = 0$

IV Calculation of DF :

Joint	Member	k	Σk	$DF = \frac{k}{\Sigma k}$
B	BA	$\frac{4EI}{3}$	3.33	0.67
	BC	$\frac{4EI}{2}$		0.33
C	CB	$\frac{4EI}{2}$	4	0.5
	CD	$\frac{4EI}{2}$		0.5

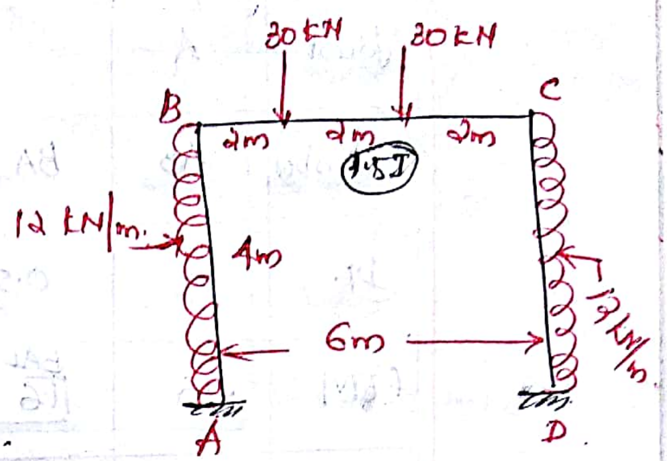
IV Moment Distribution Method :

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
DF	—	0.67	0.33	0.5	0.5	—
FEM	-12	BAL M = -15 -12	27	BAL M = -27 27	0	0
DM	0	-10.05	-4.95	-13.5	-13.5	0
COM	-5.025	BAL M = 6.75 0	-6.75	BAL M = 2.475 -2.475	0	-6.75
DM	0	4.52	2.2275	1.2375	1.2375	0
COM	2.26	BAL M = -0.6187 0	0.6187	BAL M = -1.113 1.113	0	0.6187
DM	0	-0.414	-0.204	-0.556	-0.556	0
COM	-0.207	BAL M = 0.278 0	-0.278	BAL M = 0.102 -0.102	0	-0.278
DM	0	0.186	0.0917	0.051	0.051	0
COM	0.093	0	0.025	0.045	0	0.025
Final Moment	-14.879	-17.758	17.78	12.814	-12.73	-6.384



Analysis of Frames Without Sway

Q.



Soln. I Fixed End Moments:

$$M_{FAB} = -\frac{wl^2}{12} = -\frac{12\times 4^2}{12} = -16 \text{ kNm}.$$

$$M_{FBA} = \frac{wl^2}{12} = 16 \text{ kNm}.$$

$$M_{FBC} = -\left[\frac{30\times 2\times 4^2}{6^2} + \frac{30\times 2^2\times 4}{6^2}\right] = -40 \text{ kNm}.$$

$$M_{FCB} = \frac{30\times 4\times 2^2}{6^2} + \frac{30\times 2^2\times 4}{6^2} = 40 \text{ kNm}.$$

$$M_{FCD} = -\frac{wl^2}{12} = -\frac{12\times 4^2}{12} = -16 \text{ kNm}.$$

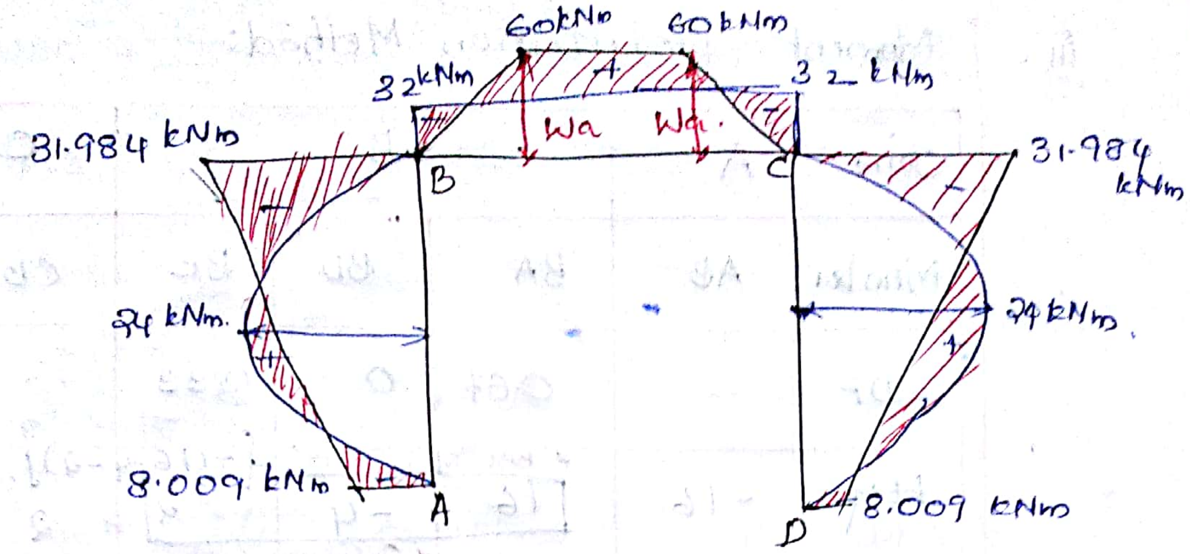
$$M_{FDC} = \frac{wl^2}{12} = 16 \text{ kNm}.$$

II Distribution Factors:

Joint	Member	k	Σk	DF = k/Σk
B	BA	$\frac{4EI}{4}$	2EI	0.5
	BC	$\frac{4E\times 1.5I}{6}$		0.5
C	CB	$\frac{4E\times 1.5I}{6}$	2EI	0.5
	CD	$\frac{4EI}{4}$		0.5

(ii) Moment Distribution Method

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
DF	–	0.5	0.5	0.5	0.5	–
FEM	-16	BAL M = +24 16	-40	BAL M = -24 40	-16	16
DM	0	12	12	-12	-12	0
COM	6	BAL M = +6 0	-6	BAL M = -6 +6	0	-6
DM	0	3	3	-3	-3	0
COM	1.5	BAL M = +1.5 0	-1.5	BAL M = -1.5 +1.5	0	-1.5
DM	0	0.75	0.75	-0.75	-0.75	0
COM	0.375	BAL M = 0.375 0	-0.375	BAL M = -0.375 0.375	0	-0.375
DM	0	0.1875	0.1875	-0.1875	-0.1875	0
COM	0.093	BAL M = 0.093 0	-0.093	BAL M = -0.093 0.093	0	-0.093
DM	0	0.0465	0.0465	-0.0465	-0.0465	0
COM	0.023	0	-0.023	0.023	0	-0.023
Final moments	-8.009	31.984	-32.007	+32.007	-31.984	8.009



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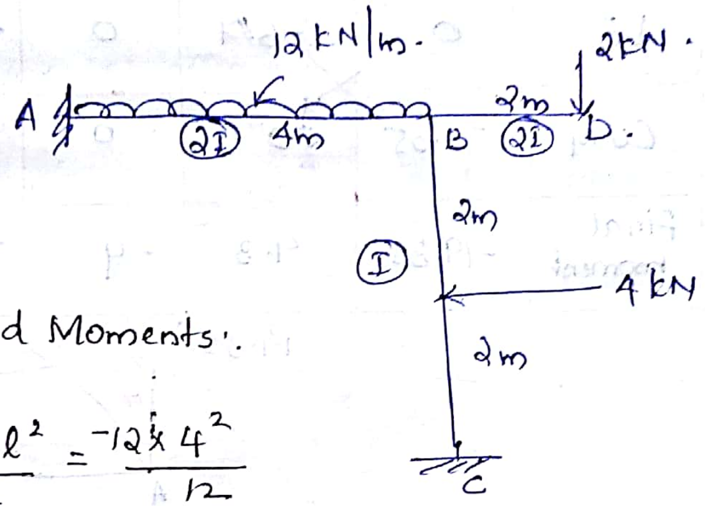


Soln. I. Fixed End Moments:

$$M_{FAB} = -\frac{wl^2}{12} = \frac{-12 \times 4^2}{12}$$

$$= -16 \text{ kNm}.$$

$$M_{FBA} = \frac{wl^2}{12} = 16 \text{ kNm}.$$

$$M_{FBC} = \frac{-Wl}{8} = \frac{-4 \times 4}{8} = -2 \text{ kNm}.$$

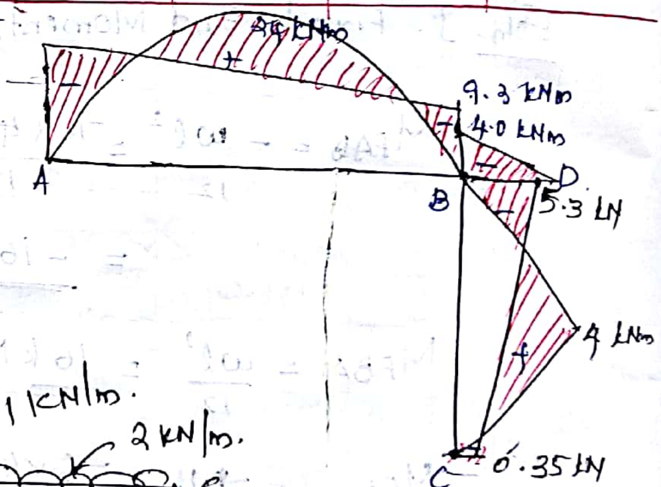
$$M_{FCB} = \frac{Wl}{8} = \frac{4 \times 4}{8} = 2 \text{ kNm}.$$

II. Distribution Factor.

Joint	Member	k	Σk	DF = $\frac{k}{\Sigma k}$
B	BA	$\frac{4E1 \times 2I}{4}$	3	0.67
	BC	$\frac{4EI}{4}$		0.33
	BD			0

iii. Moment Distribution Method:

Joint	A	B			C	D
Member	AB	BA	BD	BC	CB	DB.
DF	—	0.67	0	0.33	—	—
FEM	-16	* BAL M = -10 {-(16-4-2)}. $\boxed{16}$	-4 * overhang	-2	2	0
DF	0	-6.7	0	-3.3	0	
COM	-3.35	0	0	0	-1.65	—
Final moment	-19.35	9.3	-4	-5.3	0.35	0



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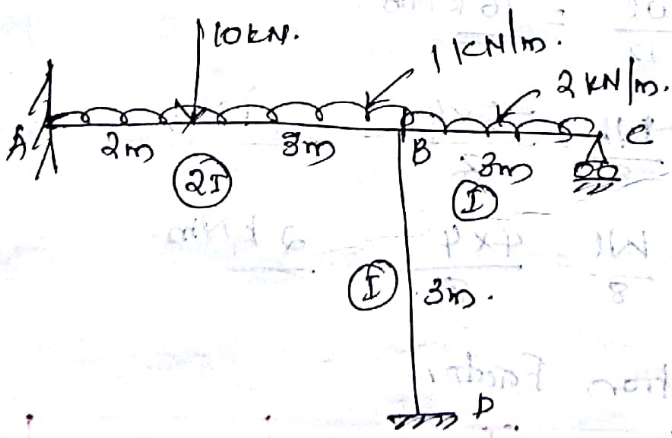


Soln-(i) Fixed End moments:-

$$M_{FAB} = -\frac{wl^2}{12} - \frac{Wab^2}{l^2} = -\frac{1\times5^2}{12} - \frac{10\times2\times3^2}{5^2} = -9.28 \text{ kNm.}$$

$$M_{FBA} = \frac{wl^2}{12} + \frac{Wa^2b}{l^2} = \frac{1\times5^2}{12} + \frac{10\times2^2\times3}{5^2} = 6.88 \text{ kNm}$$

$$M_{FBC} = \frac{-wl^2}{12} = \frac{-2\times3^2}{12} = \underline{\underline{-1.5}}\ \text{kNm}.$$

$$M_{FCB} = \frac{wl^2}{12} = \underline{\underline{1.5}}\ \text{kNm}.$$

$$M_{FBD} = 0.$$

$$M_{FCD} = 0.$$

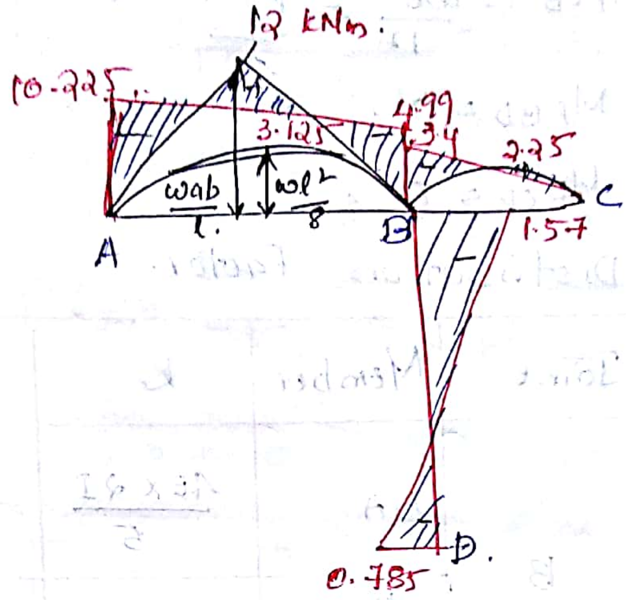
II Distribution Factor.

Joint	Member	k	Σk	$DF = \frac{k}{\Sigma k}$
B	BA	$\frac{4E\times2I}{5}$	$\frac{59}{15}EI$	0.41
	BD	$\frac{4EI}{3}$		0.34
	BC	$\frac{3EI}{3}$		0.25
C	CB			1

III Moment Distribution Method.

Joint	A	B			C	D
Member	AB	BA	BD	BC	CB	DB
DF	0	0.41	0.34	0.25	1	0
FEM	−9.28	6.88	0	−1.5	BAL M = −1.5 1.5	0
Bal.					−1.5×1 −1.5	
COM				0.75		
Final moment	−9.28	BAL M = −4.63 $\boxed{6.88}$	$\boxed{0}$	$\boxed{-2.25}$	0	0
DM	0	−1.89	carryover to DB −1.57	−1.15	0	0
COM	−0.945	0	0	0	[illegible]	−1.57/2 −0.785

Final Moment	-10.225	4.99	-1.57	-3.4	0	-0.785



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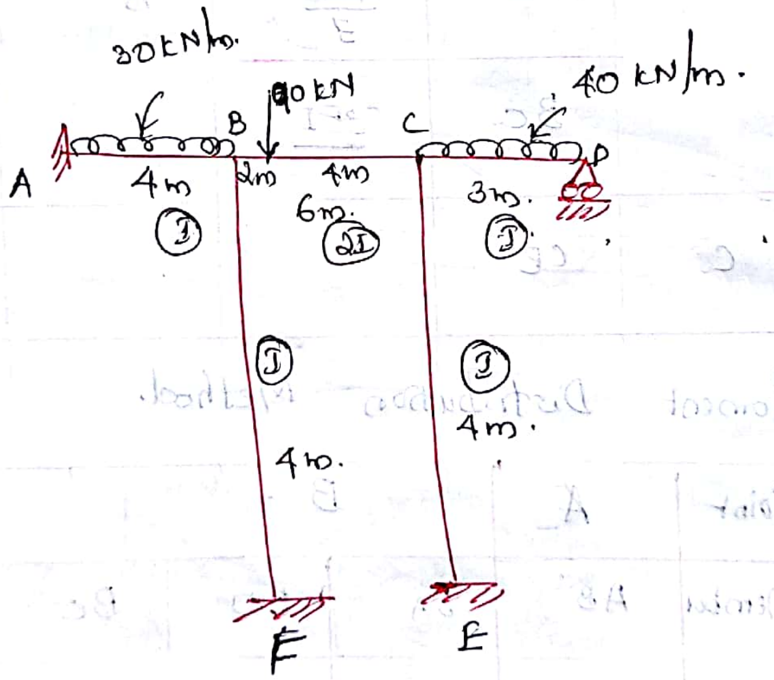


Soln. I. Fixed End moments.

$M_{FAB} = -\frac{wl^2}{12} = \frac{-30\times4^2}{12} = -40$ kNm.

$M_{FBA} = \frac{wl^2}{12} = 40$ kNm.

$M_{FBC} = -\frac{Wab^2}{l^2} = \frac{-90\times2\times4^2}{6^2} = -80$ kNm.

$M_{FCB} = \frac{Wa^2b}{l^2} = \frac{Wa^2b}{l^2} = \frac{90\times2^2\times4}{6^2} = 40$ kNm.

$M_{FCD} = \frac{-wl^2}{12} = -\frac{40 \times 3^2}{12} = -30 \text{ kNm}.$

$M_{FDC} = \frac{wl^2}{12} = 30 \text{ kNm}.$

$M_{FBF} = M_{FFB} = M_{FCE} = M_{FEC} = 0 \text{ kNm}.$

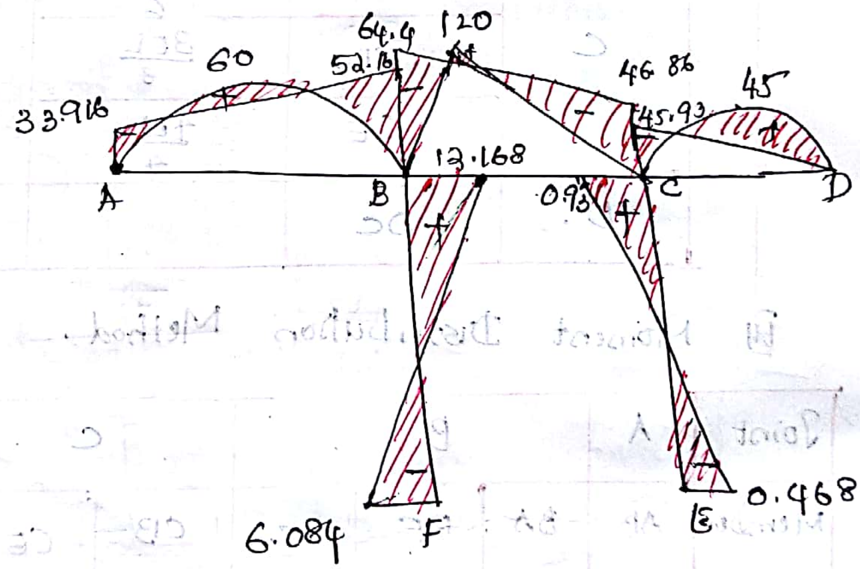
ii. Distribution Factor.

Joint	Member	k	Σk	$DF = \frac{k}{\Sigma k}$
B	BA	$\frac{4EI}{4}$	$\frac{10}{3}EI$	0.3
	BC	$\frac{4E \times 2I}{6}$		0.4
	BF	$\frac{4EI}{4}$		0.3
C	CB	$\frac{4E \times 2I}{6}$	$\frac{10}{3}EI$	0.4
	CD	$\frac{3EI}{3}$		0.3
	CE	$\frac{4EI}{4}$		0.3
D	DC			1

iii Moment Distribution Method.

Joint	A	B			C			D	E	F
Member	AB	BA	BF	BC	CB	CE	CD	DC	EC	FB.
DF	0	0.3	0.3	0.4	0.4	0.3	0.3	1	0	0
FEM	-40	40	0	-80	40	0	-30	Bal M = -30 30	0	0
Bal								-30		
COM							-15			
Final moment	-40	40	Bal M = 40 0	-80	40	Bal M = 5 0	-45	0	0	0
DM	0	12	12	16	2	1.5	1.5	0	0	0
COM	6	0	Bal M = -1 0	1	8	Bal M = -8 0	0	-	0.75	6

DM	0	-0.3	-0.3	-0.4	-3.2	-2.4	-2.4	—	0	0
		Bal M = 1.6			Bal M = 0.2					
COM	-0.15	$\boxed{0}$	$\boxed{0}$	$\boxed{-1.6}$	$\boxed{-0.2}$	$\boxed{0}$	$\boxed{0}$	—	-1.2	-0.15
DM	0	0.48	0.48	0.64	0.08	0.06	0.06	—	0	0
		Bal M = -0.04			Bal M = -0.32					
COM	0.24	$\boxed{0}$	$\boxed{0}$	$\boxed{0.04}$	$\boxed{0.32}$	$\boxed{0}$	$\boxed{0}$	—	0.03	0.24
DM	0	-0.012	-0.012	-0.016	-0.128	-0.096	-0.096	—	0	0
COM	-6×10^{-3}	0	0	-0.064	-8×10^{-3}	0	0	—	-0.048	-16×10^{-3}
Final moment	-33.916	52.168	12.168	-64.4	46.864	-0.936	-45.936	0	-0.468	6.084



SWAY ANALYSIS

If there is no symmetry in geometry, in loading & if there is no support at the beam level, there will be lateral movement of the column which is called as sway. The procedure explained in fig, will not take care of such situations. The procedure to be followed for the analysis of frames with sway given as:

(i) Assume the sway in the frame shown in fig, is prevented by giving external support at beam level. Carry out analysis as explained in the fig. This is called Non-sway analysis. Considering the free body diagram of column find Hori. forces developed at supports. Then consider the horizontal equilibrium of the entire system to get force 's' developed at additional support assumed at beam level.

(ii) Actually there is no support at beam level & hence 's' is the sway force moving the beam laterally as shown in the fig. for the given sway force - It is difficult to find the end moments developed. Hence the following procedure is adopted.

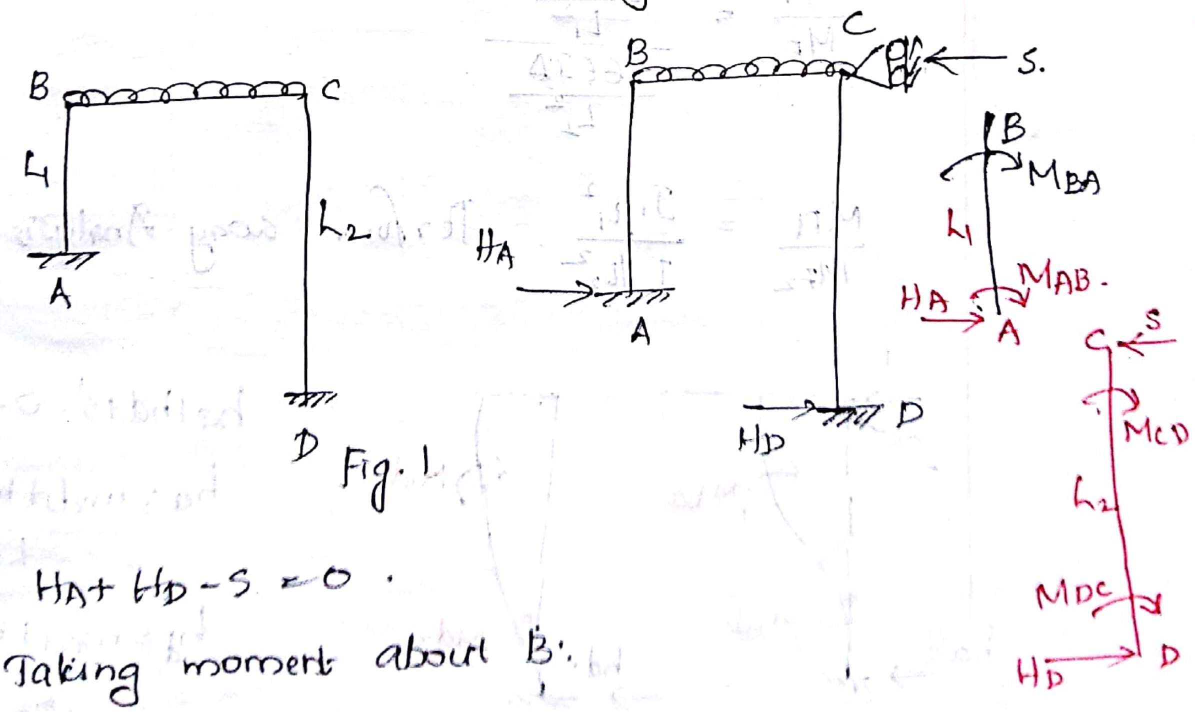


Fig. 1

$H_A + H_D - S = 0$.

Taking moment about B.

$- H_A L_1 + M_{AB} + M_{BA} = 0$.

$$H_A = \frac{M_{AB} + M_{BA}}{L_1}$$

Taking moment about C:

$$-H_D L_2 + M_{CD} + M_{DC} = 0$$

$$H_D = \frac{M_{DC} + M_{CD}}{L_2}$$

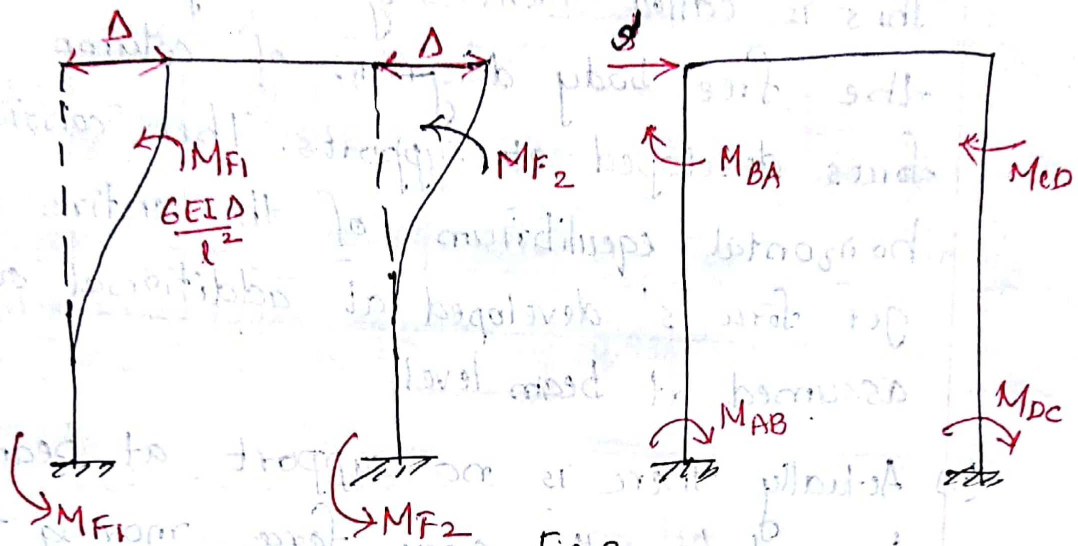
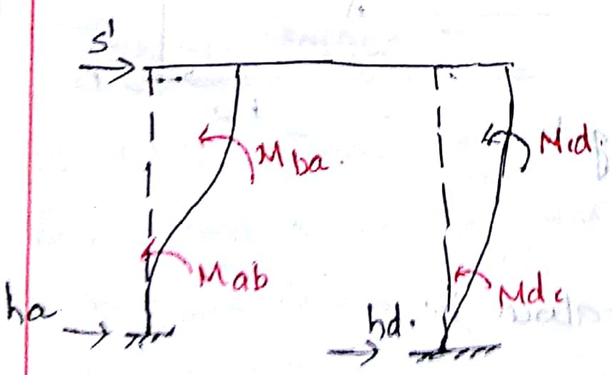


Fig. 2

where $M_F = -\frac{6EI_1\Delta}{L^2}$

$$\frac{M_{F1}}{M_{F2}} = \frac{\frac{-6EI_1\Delta}{L_1^2}}{\frac{-6EI_2\Delta}{L_2^2}}$$

$$\frac{M_{F1}}{M_{F2}} = \frac{I_1/L_1^2}{I_2/L_2^2}$$ Perform sway Analysis



$h_a + h_d + s' = 0$

$h_a = \frac{m_{ab} + m_{ba}}{L_1}$

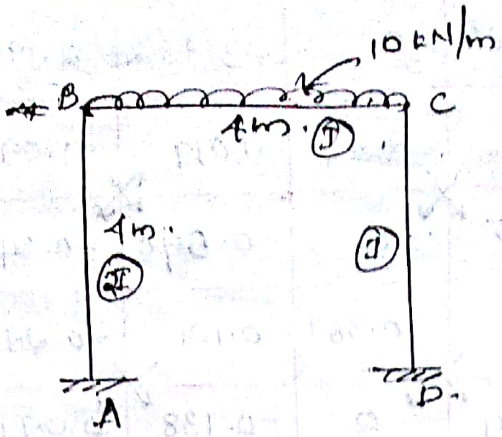
$h_d = \frac{m_{cd} + m_{dc}}{L_2}$

$k = \frac{s}{s_1}$

Hence,

$$\boxed{\text{Actual Moment} = M_{non\text{-}sway} + k \cdot M_{sway}}$$

Q.



Soln Fixed End moments.

I. $M_{FAB} = 0 = M_{FBA}$

$$M_{FBC} = -\frac{wl^2}{12} = -\frac{10\times4^2}{12} = -13.33 \text{ kNm}$$

$$M_{FCB} = \frac{wl^2}{12} = 13.33 \text{ kNm.}$$

$M_{FCD} = M_{FDC} = 0.$

II. Calculation of Distribution factor.

Joint	Member	k	Σk	$DF = \frac{k}{\Sigma k}$
B	BA	$\frac{4EI\times2}{4}$	3	0.64
	BC	$\frac{4EI}{4}$		0.33
C	CB	$\frac{4EI}{4}$	2	0.5
	CD	$\frac{4EI}{4}$		0.5

III Moment Distribution Method.

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
DF	0	0.67	0.33	0.5	0.5	0
FEM	0	Bal M = 13.33 0	-13.33	Bal M = -13.33 13.33	0	0
DM	0	8.93	4.39	-6.665	-6.665	0

COM	4.465	0	-3.332	2.195	0	-3.332
DM	0	2.23	1.099	-1.0975	-1.0975	0
COM	1.115	0	-0.548	-0.549	0	-0.548
DM	0	0.367	0.181	-0.275	-0.275	0
COM	0.184	0	-0.138	0.091	0	0.138
DM	0	0.092	0.045	-0.045	-0.045	0
COM	0.046	0	-0.022	0.022	0	-0.022
Final moment	5.81	11.619	-11.65	8.105	-8.08	-4.04

To find sway force 'S':

$$H_A = \frac{M_{AB} + M_{BA}}{L_1}$$

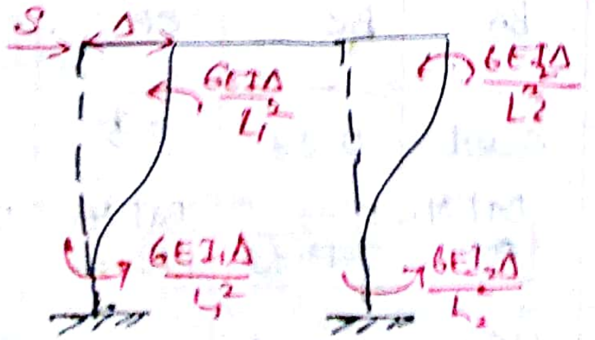
$$= \frac{5.8 + 11.6}{4} = \underline{\underline{4.35 \text{ kN}}}.$$

$$H_D = \frac{M_{CD} + M_{DC}}{4}$$

$$= \frac{-8.1 - 4.06}{4} = \underline{\underline{-3.04 \text{ kN}}}.$$

$$H_A + H_D - S = 0.$$

$$S = H_A + H_D = 4.35 - 3.04 = \underline{\underline{1.31 \text{ kN}}}.$$



Note: $\frac{6EI\Delta}{L^2}$ applied to columns only.

$$\frac{M_{F_1}}{M_{F_2}} = \frac{\dfrac{-6EI\Delta}{l_1^2}}{\dfrac{-6EI_2\Delta}{l_2^2}} = \frac{-I_1/L_1^2}{-I_2/L_2^2} = \frac{-2I/4^2}{-I/4^2} = \frac{-2}{-1}$$

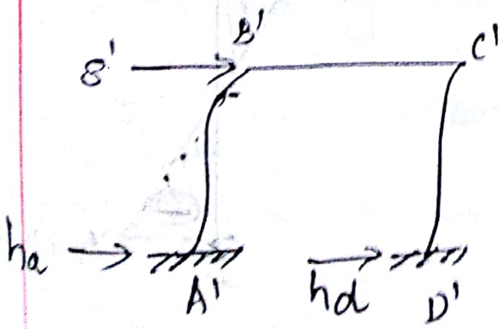
$$= -2 \times \frac{10}{10} = \frac{-20}{-10}$$

Multiply & divide by same factor

$M_{F_1} = -20$ kNm.

$M_{F_2} = -10$ kNm.

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
DF	0	0.67	0.33	0.5	0.5	0
FEM	Anticlockwise -ve -20	Bal M = 20 $\boxed{-20}$	$\boxed{0}$	Bal M = 10 $\boxed{0}$	$\boxed{-10}$	-10
DM	0	13.4	6.6	5	5	0
COM	6.7	0	2.5	3.3	0	2.5
DM	0	-1.675	-0.825	-1.65	-1.65	0
COM	-0.84	0	-0.825	-0.413	0	-0.825
DM	0	0.55	0.27	0.207	0.207	0
COM	0.28	0	0.104	0.135	0	0.104
DM	0	-0.069	-0.034	-0.068	-0.068	0
COM	-0.035	0	-0.034	-0.017	0	-0.034
Final moment	-13.895	-7.794	7.76	6.49	-6.51	-8.26



$$h_a = \frac{m_{ab} + m_{ba}}{l_1} = \frac{-13.895 + -7.794}{4} = -5.42\text{ kN}.$$

$$h_d = \frac{m_{cd} + m_{dc}}{l_2} = -6.51 - 8.26 = -3.69\text{ kN}$$

$h_a + h_d + s' = 0$ {equilibrium}

$-5.42 - 3.69 + s' = 0$.

$s' = 5.42 + 3.69 = \underline{9.11 \text{ kN}}$.

$\therefore$ Correction factor(K) $= \frac{s}{s'} = \frac{1.31}{9.11}$

$= \underline{0.143}$.

$\therefore$ Final moment (actual moment)

$= M_{non\text{-}sway} + K M_{sway}$

Final moments :

$M_{AB} = 5.81 + 0.143 \times -13.895 = \underline{3.823} \text{ kNm}$.

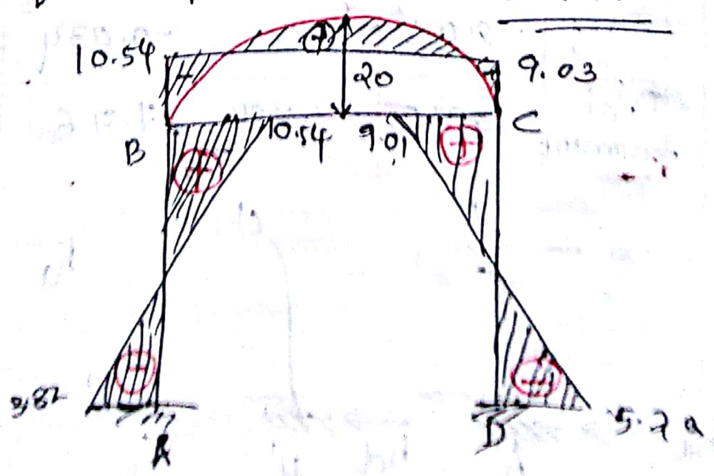
$M_{BA} = 11.619 + 0.143 \times -7.794 = \underline{10.547} \text{ kN-}$

$M_{BC} = -11.65 + 0.143 \times 7.76 = \underline{-10.54} \text{ kNm}$.

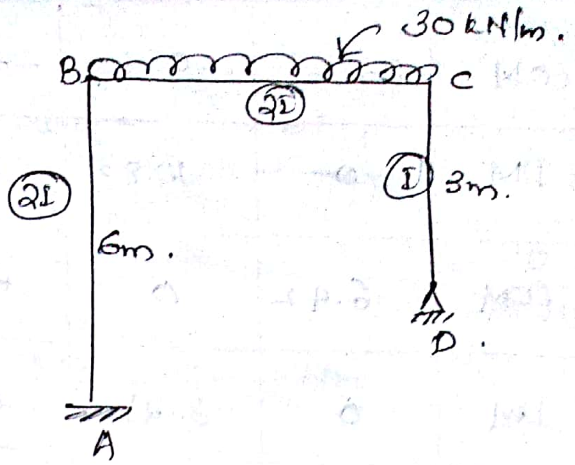
$M_{CB} = 8.105 + 0.143 \times 6.49 = \underline{9.033} \text{ kNm}$.

$M_{CD} = -8.08 + 0.143 \times -6.51 = \underline{-9.012} \text{ kNm}$.

$M_{DC} = -4.04 + 0.143 \times -8.26 = \underline{-5.221} \text{ kNm}$.



Q.



Soln. (I) Fixed End Moments.

$M_{FAB} = M_{FBA} = 0.$

$M_{FCD} = M_{FDC} = 0.$

$M_{FBC} = \frac{-wl^2}{12} = \frac{-30 \times 6^2}{12}$

$= \underline{-90 \text{ kNm}}$

$M^F_{CB} = \frac{wl^2}{12} = \underline{90 \text{ kNm.}}$

II. Calculation of Distribution factor

Joint	Member	k	Σk	DF = k/ΣF
B.	BA	$\frac{4EI \times 2}{6} = \frac{4EI}{3}$	$\frac{8}{3}EI$	0.5
	BC	$\frac{4EI \times 2}{6}$		0.5
C	CB.	$\frac{4E \times 2I}{6}$	$\frac{7}{3}EI$	0.57
	CD	$\frac{3EI}{3}$		0.43
D.	DC	$\frac{4EI}{3}.$	–	1

III. Moment Distribution Method.

Joint	A	B		C		D.
Member	AB	BA	BC	CB	CD	DC
DF	0	0.5	0.5	0.57	0.43	1
FEM	0	0	−90	90	0	0
DM	–	45	45	−51.3	−38.7	–

D hinge support bcz of DC = 0 ∴ no need of balancing

COM	22.5	0	-25.65	22.5	0	—
DM	0	12.83	12.83	-12.83	-9.68	—
COM	6.42	0	-6.42	6.42	0	—
DM	0	3.21	3.21	3.66	2.76	—
COM	1.61	0	1.83	1.61	0	—
DM	0	-0.92	-0.92	-0.92	-0.69	—
COM	-0.46	0	-0.46	-0.46	0	—
DM	0	0.23	0.23	0.262	0.198	—
COM	0.12	0	0.131	0.12	0	—
DM	0	-0.066	-0.066	-0.068	-0.052	—
COM	-0.033	0	-0.034	-0.033	0	—
Final Moment	31.16	62.3	-62.3	51.91	-51.91	0

Non-Sway Analysis:

$H_A + H_D - S = 0$

$H_A = \frac{M_{AB} + M_{BA}}{L_1}$

$= \frac{31.16 + 62.3}{6}$

$= 15.576$ kN.

Note: If D is roller support, then equilibrium eqn will be $H_A - S = 0$ ∵ no horizontal reaction for roller support.

$H_D = \frac{M_{CD} + M_{DC}}{L_2}$

$= \frac{-51.91 + 0}{3} = -17.303$ kN

∴ $15.576 - 17.303 - S = 0$

$S = -1.727$ kN.

Sway Analysis :-

$$\frac{M_{F1}}{M_{F2}} = \frac{\frac{-6EI_1\Delta}{L_1^2}}{\frac{-6EI_2\Delta}{L_2^2}} = \frac{-I_1/L_1^2}{-I_2/L_2^2} = \frac{-2I/6^2}{-I/3^2} = \frac{-1}{-2} \times \frac{10}{10}$$

$$= \frac{-10}{-20}$$

$\therefore M_{F1} = -10 \text{ kNm}$. $M_{F2} = -20$ kNm.

Joint	A	B		C		D.
Member.	AB	BA	BC	CB	CD	DC
DF	0	0.5	0.5	0.57	0.43	1
FEM	−10	−10	0	0	−20	−20
Bal.						20
COM					10	
Moment	−10	−10	0	0	−10	0
DM	0	5	5	5.7	4.3	—
COM	2.5	0	2.85	2.5	0	—
DM	0	−1.43	−1.43	−1.431	−1.075	—
COM	−0.715	0	−0.715	−0.715	0	—
DM	0	0.358	0.358	0.407	0.307	—
COM	0.179	0	0.204	0.179	0	—
DM	0	−0.102	−0.102	−0.102	−0.077	—
COM.	−0.051	0	−0.051	−0.051	0	—
Final moment	−8.09	−6.17	6.912	6.49	−6.54	0

$h_a + h_d + s' = 0$.

$$h_a = \frac{m_{ab} + m_{ba}}{l_1}$$

$$= \frac{-8.09 - 6.17}{6}$$

$$= \underline{\underline{-2.38\ kN}}.$$

$$h_d = \frac{m_{cd} + m_{dc}}{l_2}$$

$$= \frac{-6.54 + 0}{3}$$

$$= \underline{\underline{-2.18\ kN}}.$$

$$-2.38 - 2.18 + s' = 0.$$

$$s' = \underline{\underline{4.56\ kN}}.$$

$\therefore$ correction factor $= \frac{s}{s'} = \frac{-1.727}{4.56} = \underline{\underline{-0.379}}$.

$\therefore$ Final moment (actual moment) $=$

$M_{non\text{-}sway} + k M_{sway}$.

Final moments.

$M_{AB} = 31.16 + -0.379 \times -8.09 = \underline{\underline{34.23}}\ kNm$.

$M_{BA} = 62.3 + -0.379 \times -6.17 = \underline{\underline{64.64}}\ kNm$.

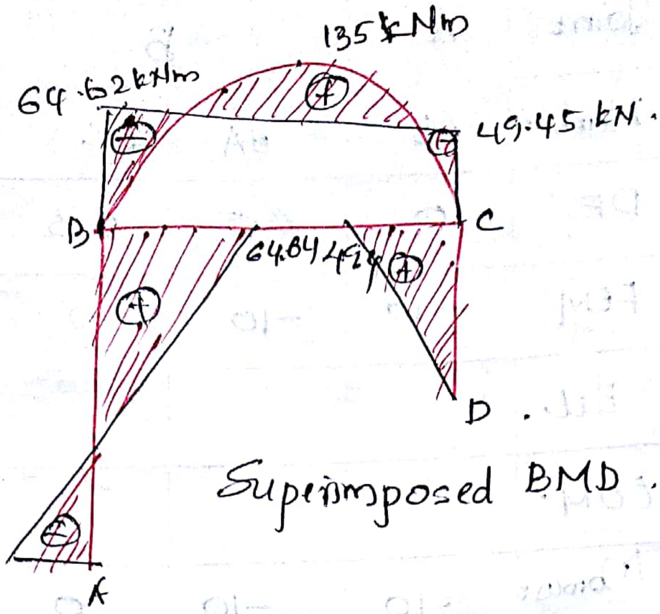
$M_{BC} = -62.37 - 0.379 \times 6.12 = \underline{\underline{-64.62}}\ kNm$.

$M_{CB} = 51.91 + -0.379 \times 6.49 = \underline{\underline{49.45}}\ kNm$.

$M_{CD} = -51.91 + -0.379 \times -6.54 = \underline{\underline{-49.43}}\ kNm$.

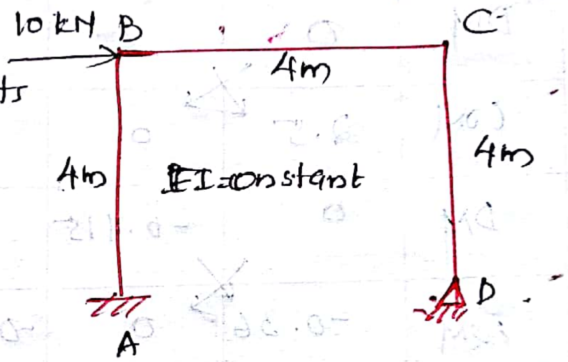
$M_{DC} = 0 - 0.379 \times 0 = \underline{\underline{0}}\ kNm$.

Scanned by CamScanner



Superimposed BMD.

Q.



Note: Here the supports A & D are different.
∴ Sway frame.

Here the horizontal load 10 kN given. Since the horizontal force acting on the structure (10 kN) is equal to the resisting force, so, we can directly go for sway analysis.

* No need to find FEM.

Distribution Factor:

Joint	Member	k	Σk	DF = k/Σk
B	BA	$\frac{4EI}{4}$	2EI	0.5
	BC	$\frac{4EI}{4}$		0.5
C	CB	$\frac{4EI}{4}$	$\frac{7}{4}EI$	0.57
	CD	$\frac{3EI}{4}$		0.43

Joint	A	B		C		D.
Member	AB	BA	BC	CB	CD	DC
DF	0	0.5	0.5	0.57	0.43	1
FEM	−10	−10	0	0	−10	−10
Bal.						10
COM.					5	
Moment	−10	−10	0	0	−5	0
DM.	0	5	5	2.85	2.15	–
COM	2.5	0	1.43	2.5	0	–
DM	0	−0.715	−0.715	−1.43	−1.08	–
COM	−0.36	0	−0.715	−0.36	0	–
DM	0	0.36	0.36	0.21	0.15	–
COM	0.18	0	0.105	0.18	0	–
DM	0	−0.05	−0.05	−0.103	−0.077	–
COM	−0.025	0	−0.052	−0.025	0	–
Final moment	−7.704	−5.41	5.36	3.82	−3.849	0

$h_a + h_d + s' = 0.$

$$h_a = \frac{m_{ab} + m_{ba}}{L}$$

$$= \frac{-7.704 - 5.41}{4}$$

$$= -3.28\ \text{kN.}$$

Do not consider horizontal force 10 kN. ∴ Here effect of sway only considered.

$$\therefore h_d = \frac{m_{cd} + m_{dc}}{l}$$

$$= \frac{-3.85 + 0}{4}$$

$$= -0.963\ \text{kN.}$$

$\therefore -3.28 - 0.963 + s' = 0$

$s' = 4.243$ kN.

$\therefore$ correction factor $= \frac{S}{S'} = \frac{10}{4.243} = 2.357$

Final moment (Actual moment) = $M_{non sway} + k M_{sway}$.

$\therefore$ Final Moments.

$M_{AB} = 0 + 2.357 \times -7.704 = -18.158$ kNm.

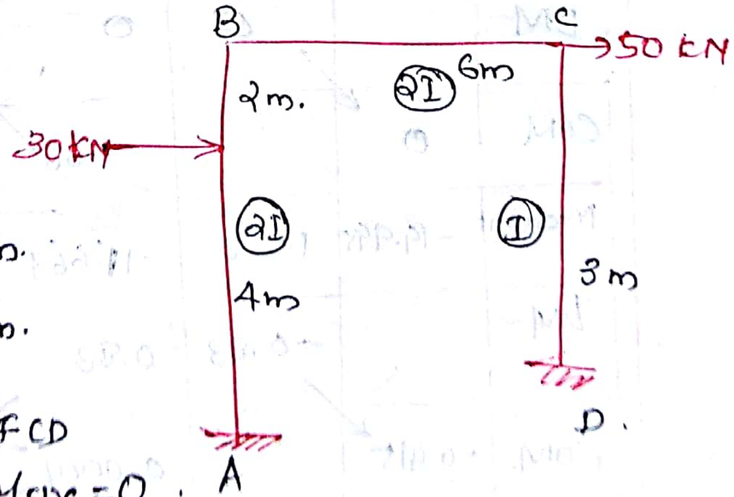
$M_{BA} = 0 + 2.357 \times -5.41 = -12.75$ kNm.

$M_{BC} = 0 + 2.357 \times 5.36 = 12.63$ kNm.

$M_{CB} = 0 + 2.357 \times 3.82 = 9$ kNm.

$M_{DC} = 0 + 2.357 \times -3.849 = -9.07$ kNm.

Q.



Sln I. Fixed End Moments.

$M_{FAB} = -13.33$ kNm.

$M_{FBA} = 26.66$ kNm.

$M_{FBC} = M_{FCB} = M_{FCD}$

$= M_{FDC} = 0$.

II. Distribution factor.

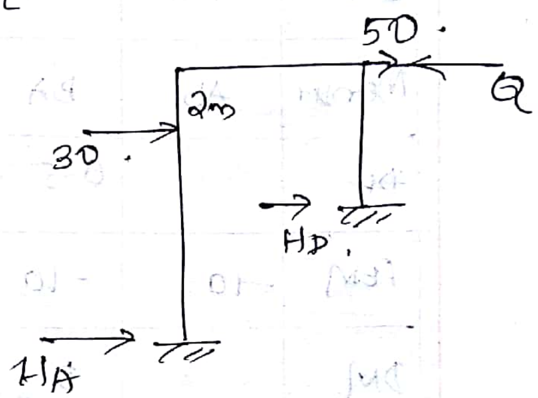
Joint	Member	k	Σk	DF = k/Σk
B	BA	$\frac{4E\times 2I}{6} = \frac{4}{3}EI$	2.66	0.5
	BC	$\frac{4E\times 2I}{6} = \frac{4}{3}EI$		0.5
C	CB	$\frac{4EI\times 2}{6} = \frac{4}{3}EI$	2.66	0.5
	CD	$\frac{4EI}{3} = \frac{4}{3}EI$		0.5

III. Moment Distribution Method.

Joint	A	B		C		D.
Member	AB	BA	BC	CB	CD	DC
DF	0	0.5	0.5	0.5	0.5	0
FEM	−13.33	26.66	0	0	0	0
DM		−13.33	−13.33			
COM	−6.665			−6.665		
Final moment	−19.995	13.33	−13.33	−6.665	0	0
DM		0	0	3.33	3.33	
COM	0		1.66	0	0	1.66
Moment	−19.995	13.33	−11.667	−3.335	3.33	+1.66
DM		−0.83	−0.83	0.0025	0.0025	
COM.	−0.415		0.00012	−0.415		−0.00012
DM		0	0	0.205	0.205	
COM	0	0	0.1025	0	0	0.025

Moment	−20.41	12.5	−12.39	−3.53	3.53	1.76
DM	0	−0.055	−0.055	0	0	0
COM	−0.027	0	0	−0.027	0	0
Final moment	−20.437	12.445	−12.5	−3.55	+3.53	1.76

To find sway force, 'Q':



$H_A + H_D + 30 = Q - 50$

$H_A + H_D + 30 + 50 = Q.$

$$H_A = \frac{M_{AB} + M_{BA} - 30 \times 2}{L}$$

$$= \frac{-20.44 + 12.5 - 30 \times 2}{6}$$

$= -11.33$ kN.

$$H_D = \frac{M_{CD} + M_{DC}}{3} = \frac{3.53 + 1.76}{3} = 1.75 \text{ kN}$$

$\therefore Q = 70.4$ kN.

Sway Analysis.

$$M_{FAB} = \frac{-6EI \times 2\Delta}{l^2} = \frac{-1}{3} EI\Delta.$$

$M_{FBA} = \frac{-1}{3} EI\Delta.$

$M_{FBC} = M_{FCB} = 0$

$M_{FCD} = M_{FDC} = \frac{-2}{3} EI\Delta.$

$$\frac{M_{FAB}}{M_{FCD}} = \frac{-1}{-2} = \frac{-10}{-20}$$

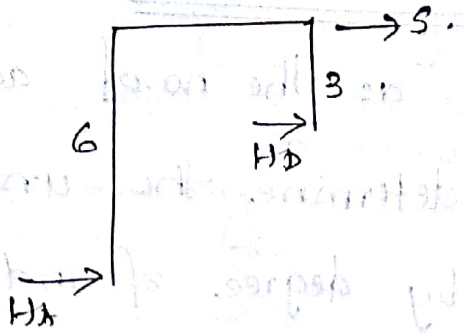
$\therefore M^F_{AB} = M_{FBA} = -10$ kNm.

$M_{FBC} = M_{FCB} = 0$.

$M_{FCD} = M_{FDC} = -20$ kNm.

Point	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
DF		0.5	0.5	0.5	0.5	
FEM	−10	−10	0	0	−20	−20
DM		5	5	10	10	
COM	2.5	0	5	2.5	0	5
Moment	−7.5	−5	10	12.5	−10	−15
DM		−2.5	−2.5	−1.25	−1.25	
COM	−1.25	0	−0.65	−1.25	0	−0.625
Moment	−8.75	−7.5	6.875	10	−11.25	−15.625
DM		0.3125	0.3125	0.625	0.625	
COM	0.156	0	0.3125	0.156	0	0.3125
Moment	−8.594	−7.187	7.5	10.781	−10.625	−15.3125
DM		−0.156	−0.156	−0.078	−0.078	
COM	−0.078	0	−0.039	−0.078	0	−0.039
Final moment	−8.672	−7.343	7.305	10.625	−10.70	−15.35

To obtain sway factor.



$\Sigma H = 0$.

$H_A + H_D + S = 0$.

$S = -(H_A + H_D)$

$H_A = \dfrac{M_{AB} + M_{BA}}{L} = \underline{2.67 \text{ kN}}$

$H_D = \dfrac{M_{DC} + M_{CD}}{L} = \underline{-8.68 \text{ kN}}$.

$\underline{S = 11.35 \text{ kN}}$.

Sway factor $= \dfrac{Q}{S} = \dfrac{70.4}{11.35} = \underline{6.202}$

∴ Final moment (actual moment) = $M_{non\text{-}sway} + k M_{sway}$

Final moments.

$M_{AB} = \underline{-74.217 \text{ kNm}}$.

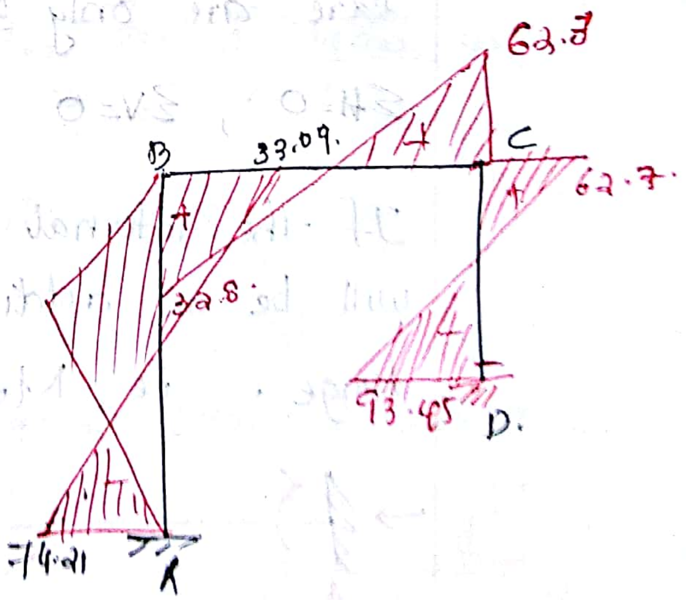
$M_{BA} = \underline{-33.09} \text{ kNm}$.

$M_{BC} = \underline{32.6} \text{ kNm}$.

$M_{CB} = \underline{62.34} \text{ kNm}$.

$M_{CD} = -\underline{62.7} \text{ kNm}$.

$M_{DC} = \underline{-93.45} \text{ kNm}$.



STATIC INDETERMINANCY.

It is defined as the no. of additional eqns required to determine the unknown reactions. It is given by degree of indeterminancy is equal to the no. of unknown reactions − no. of static eqbm eqns.

The unknown reaction components @

Roller = 1 ↑.

Hinged = 2 → ↑

Fixed = 3 → ↺ ↑

Hence the total no. of unknowns are found in the entire beam structure.

There are only 3 independent eqns.

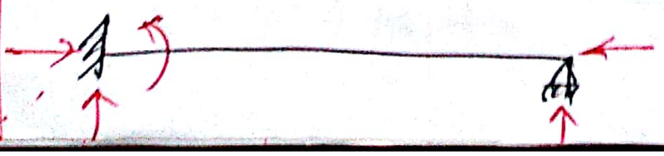
$\Sigma H = 0$, $\Sigma V = 0$ & $\Sigma M = 0$.

If the internal hinges are provided, there will be an additional eqbm eqns for each hinge. ie, Moment at hinge = 0.

R = 6

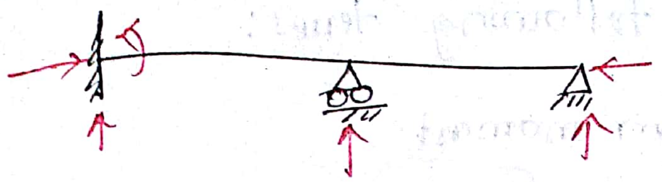
n = 3.

i = 6 − 3 = <u>3</u>.



R = 5 n = 3

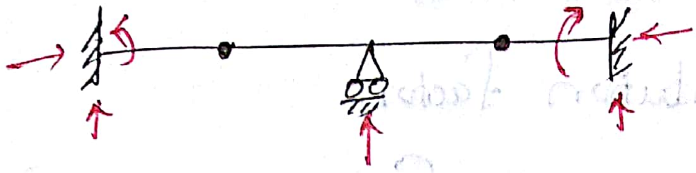
i = 5 − 3 = 2



$R = 6$

$n = 3$.

$i = 6 - 3 = \underline{3}$.



hinges.

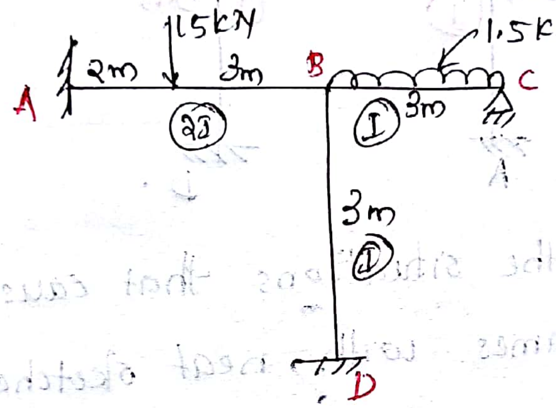
$R = 7$

$n = 5 \quad (3 + 2)$

$i = 7 - 5 = \underline{2}$.

UNIVERSITY QUESTIONS.

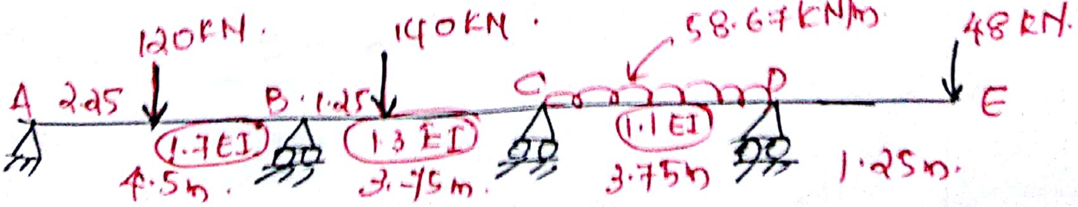
1. Analyse the 2D frame shown in fig. using moment distribution method. Draw BMD.



2. Derive expressions for stiffness at the near-end and carry over factor for a beam with hinged far-end.

3. Explain moment distribution method.

4. Analyse the continuous beam shown in fig. by moment distribution method. Draw BMD & SFD.



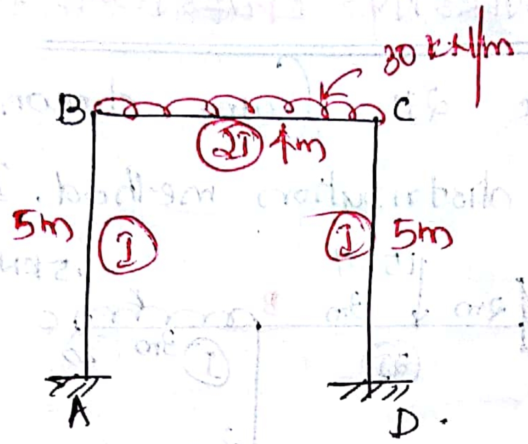
5. Define the following terms:

(i) Carry over moment.

(ii) Carry over factor.

(iii) Distribution factor.

6. Analyse the rigid frame shown in fig. by moment distribution method and draw the BMD.



7. List out the situations that causes sway in portal frames with neat sketches.

8. Explain the procedure to be followed for the analysis of rigid frames with sway by method of moment distribution.